# Monetary Policy with Financially-Constrained and Unconstrained Firms

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**Abstract**

Literature so far has studied the transmission and optimal design of monetary policy in setups with either only financially-unconstrained firms or with only financially-constrained firms. This paper analyses these questions in an extension of the basic New Keynesian model with both types of firms, and yields a number of novel theoretical insights. (i) The interactions of the two types of firms on input and output markets activate a new transmission channel (the "spillover channel"). Because of this new channel, (ii) aggregate output does not necessarily respond more strongly to monetary policy, and (iii) the optimal design of monetary policy does not necessarily change when the share of constrained firms is higher (contrary to the financial accelerator intuition). The model is used to discuss the responses to monetary policy of financially constrained and unconstrained firms in the UK.

**Keywords:** New Keynesian model, financially-constrained firms, firm heterogeneity, working-capital credit, monetary policy transmission, optimal monetary policy

**JEL Class.:** E2 – E3 – E4.

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# 1 Introduction

As monetary policy affects financially-constrained and unconstrained firms to different degrees (figure 1), its overall impact ought to vary with the share of constrained firms in the economy. But it is not clear how. The theoretical literature on the transmission channels and design of monetary policy is ill-equipped to address this question because it relies on models that feature either constrained firms or unconstrained firms —but not both. In this paper, I enrich the basic New Keynesian model by allowing for both types of firms, and use it the revisit the literature.

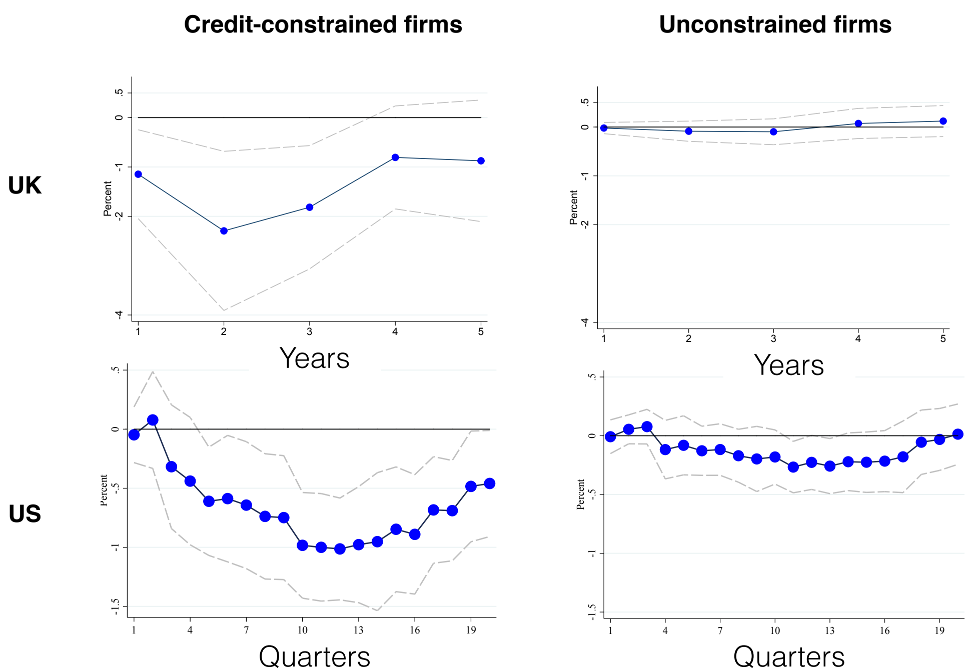



Figure 1: Estimated heterogenous responses of investment to a transitory monetary tightening: financially-constrained *versus* unconstrained firms in the UK (top panels) and the US (bottom panels)
*Source:* Cloyne et al. (2018)

The proposed model is a heterogenous-firm version of the basic NK framework where a share of firms face collateral-constraints, while the others do not face any financing constraints. All firms finance physical and working capital with equity ("net worth") and collateralized debt. Physical capital in fixed aggregate supply ("real estate") serves both as production input and collateral[1].

[1]Real-estate collateral is the working hypothesis of empirical studies on the macroeconomic effects of firms' collateral constraints (Fort et al. (2013), Adelino et al. (2015) for the US, Kleiner (2015), Bahaj et al. (2018) for the UK, Gan (2007) and Lian and Ma (2018) for Japan, and Banerjee and Blickle (2016) and Schmalz et al. (2017) for European

Firms are heterogenous on one dimension: some of them have high net worth and use it to finance production, while others have low net worth and thus need to rely on (collateralized) debt. The latter end up credit-constrained in equilibrium. I hereafter call these firms for short "constrained". They can be interpreted, for instance, as young firms which lack the net worth and/or performance records required for easy access to credit markets. The model equals the basic NK setup with nominal rigidities à la Rotemberg (1982) on all other dimensions.

Firms' collateral constraints activate a number of new monetary policy transmission mechanisms. In the basic version without financial frictions, production by all firms is affected via the aggregate demand channel: as individual demand schedules decline in response to a rise in the interest rate, firms unequivocally reduce both prices and output levels. In the presence of credit frictions however, a monetary tightening affects production by constrained firms via additional mechanisms in opposite directions. On the one hand, it affects production by constrained firms *negatively* by pushing downwards collateral values and upwards the real value of nominal debt (the standard balance-sheet and debt deflation channels). On the other hand, it affects it *positively* because it reduces prices of inputs financed against collateral (*an input price channel* not considered so far in the literature). Effects on constrained firms further spill over to unconstrained ones via input and output markets, either enhancing or dampening the direct effect of monetary policy on the latter. These "spillovers" constitute a seconds new channel of monetary policy transmission which has been overlooked in the literature.

Other three interesting conclusions emerge from the analysis of the effect of firms' credit frictions on aggregate transmission. First, contrary to conventional wisdom, given the opposing nature of mechanisms affecting output of constrained firms and the existence of spillovers, credit frictions may both amplify or dampen the response of aggregate output to monetary policy. A strong balance-sheet channel favors amplification, whereas a strong input-price channel favors dampening. Second, because of spillovers (and hence, general equilibrium effects), observing constrained firms responding more on average (as for instance in the UK or the US) does not necessarily imply that credit frictions significantly amplify the macroeconomic response. This is because, while constrained firms are severely affected and respond more than a firm in a world without financial frictions, the unconstrained ones relatively benefit, and their activity reduces less than in the absence of

countries).

credit frictions. Third, since monetary policy affects not only aggregate demand but also the supply of constrained firms, it may have an unexpected impact on prices. Specifically, a contractionary monetary policy may depress supply strongly enough together with demand to induce a rise in the price level in equilibrium (instead of a decline as in the basic model).

The changes in monetary transmission have also implications for the optimal design of monetary policy. Given its empirical relevance, I focus exclusively on the case of a dominant balance-sheet channel. I find that spillovers play an important role and hence the optimal policy response is very different from the weighted mean of the two polar limiting cases (with only unconstrained, or with only constrained firms). Furthermore, according to the analysis, the strength of the balance-sheet channel affects decisively the optimal response to non-financial shocks. Specifically, the stronger this channel, the weaker the reduction of the policy rate under optimal policy in response to negative demand and positive technology shocks. This is because in the presence of constrained firms and of a dominant balance-sheet channel, the decline in the policy rate is associated not only to an upward shift in aggregate demand, but also in aggregate supply. Thus, relatively to a credit-frictionless environment, in response to a negative demand shock or to a positive technology shock, a monetary loosening induces additional deflationary pressures via its supply side effects. Thus, when the balance-sheet channel is strong, contrary to the credit-frictionless benchmark, a decline in the policy rate that allows to close the output gap, may be associated to inefficient deflationary pressures in equilibrium.

In response to an adverse financial shock, modeled as an exogenous reduction in the pledgeability of capital as collateral, I find that the policy rate declines so as to prop up collateral asset prices under optimal monetary policy. This decline is aimed at (partially) counteracting the effect of the shock. A similar result is obtained by Andres et al. (2013) based on the setup in Iacoviello (2005) which features a strong balance-sheet channel in the context of collateral constraints, as well as by De Fiore and Tristani (2013) in the context of costly-state verification for a shock to constrained firms's net worth. The latter analyses however do not take into account the transmission spillovers between the two types of firms and the effect of steady-state distortions. Thus, relatively to them, in the current heterogenous firm setup we can also see how the decline in the policy rate further induces unconstrained firms to expand production, and to push output above its inefficient steady-state level.

In the second part of the paper, I use firm-level data to test the theoretical predictions of the model. For this purpose, I follow Cloyne et al. (2018) and I use firm balance-sheet data from the WorldScope database for the UK. Constrained firms are defined as young firms which do not distribute dividends. I estimate the responses of constrained versus unconstrained firms to monetary policy using an instrumental variable version of the local projection method developed by Jorda et al. (2019), with the interest rate instrumented by high frequency monetary policy shocks from Gerko and Rey (2015). Theoretical predictions of the model regarding monetary policy transmission are consistent with the data. In particular, the model predicts that all else equal, a lower pledgeability of capital or a lower liquidity ratio are associated to a stronger response of constrained firms to monetary policy in the conventional direction, which is corroborated by the data. Moreover, at least for a subset of constrained firms, a monetary tightening appears to steer their activity in an unconventional positive direction. This happens for those with only short-term debt (used as a proxy for working capital credit) and a low fraction of tangible capital.

In the third part of the paper, I discuss how the model can be used for policy analysis. Given its relevance at the moment, I focus on the particular case of a low interest rate environment. I start by interpreting the estimated impact of monetary policy on constrained versus unconstrained firms in the UK through the lenses of the model. Estimations show that constrained firms reduce their activity stronger than unconstrained ones in response to a monetary tightening. These results are consistent with a dominant balance-sheet channel. Specifically, the increase in the policy rate reduces collateral asset values, and this makes constrained firms cut strongly production. The strong adverse effects on constrained firms have positive spillovers for unconstrained ones, and these positive spillovers partially counteract the negative effects of the monetary tightening on unconstrained firms. Hence, the latter respond only mildly in equilibrium.

I then show how in economies with strong dominant balance-sheet channels such as the UK (and also the US and most likely the Euro Area), the ZLB on the policy rate hurts constrained firms, and may benefit unconstrained ones. Specifically, in the absence of the ZLB, constrained firms are more negatively affected than unconstrained ones in response to a monetary tightening, but more positively affected in response to a loosening. Thus, in the absence of the ZLB, the relatively gains and losses for these firms compensate over the business cycle. Since the ZLB limits the more positive effects of monetary policy when a decline in the policy rate is warranted, but the more adverse effects remain unchanged, on average over the business cycle, constrained firms end up particularly hurt. Furthermore, because of spillover effects, unconstrained firms benefit. Specifically, since constrained firms are affected less positively by a cut in interest rates, their negative spillovers to unconstrained ones via input and output markets are also lower. As a result, monetary policy has stronger net positive effect on unconstrained firms in times when a cut in the policy rate is needed. Thus, despite the stronger decline in aggregate activity when the ZLB binds, unconstrained firms may not be affected by the latter in equilibrium, or they may even end up producing more.

Hereafter, section 2 reviews related literature, section 3 describes the model, section 4 analyses monetary policy transmission, and section 5 focuses on optimal design. The paper concludes by discussing future extensions.

## 2 Relation to the literature

The paper is related to three main strands of literature. The first one is the theoretical literature on the transmission and optimal design of monetary policy in the absence of financial frictions (Galí (2015), Woodford (2003)). In this world, monetary policy transmits to the economy by shifting its aggregate demand channel. It can thus always insulate it from the (welfare loss) effects of demand shocks, which is equivalent to varying the policy rate so as to achieve zero inflation in equilibrium. By contrast, in response to technology shocks, this literature finds that monetary policy should optimally target a composite measure of price and wage inflation. This is because the efficient real wage varies, and this variation is approached via a (costly) adjustment in both sticky prices and wages. Finally, financial shocks play no role in this frameworks.

The second strand of literature is the one studying the implications of firms' financial frictions for monetary policy[2]. With two exceptions (Gilchrist et al. (2017) and Ottonello and Winberry (2018)), all models used so far feature only financially-constrained firms (Iacoviello (2005), Carlstrom et al. (2010), Andres et al. (2013) for collateral constraints, De Fiore and Tristani (2013), Faia and Monacelli (2007), Hansen (2018) and Fendoglu (2014) for other types of financial frictions). The two exceptions are models built to rationalize particular stylized facts identified in US data, namely (i) why financially-constrained firms increased prices during the recent financial crisis, while the

[2]An extensive strand of theoretical literature starting with Kiyotaki and Moore (1997) studies how such financial frictions may alter macroeconomic dynamics within flexible-price ('real') frameworks. These models however do not consider monetary policy.

unconstrained decreased them, and (ii) why low-leverage firms respond more to monetary policy. Relatively to these two models which are tailored to deal with very specific questions, the model in this paper is more general. Thus, *inter-alia*, it can be used also to rationalize the two particular patterns. Specifically, the first pattern emerges as well in the current monopolistic competition setup (in the absence of customer-based markets as assumed by the former paper), whereas the second arises to the extent that low-leveraged firms are financially-constrained firms with low collateral.

Furthermore, papers in this literature have either considered working capital credit or investment credit, whereas the current analysis considers both[3]. The inclusion in the model of both types of credit enables to reconcile seemingly contradictory findings of previous studies on the implications of financial frictions for the macroeconomic effects of monetary policy. For instance, collateral constraints strongly amplify the effect of monetary policy on output in Iacoviello (2005) and Andres et al. (2013) (which considered the case of investment credit), whereas they hardly have any effect in Carlstrom et al. (2010) or De Fiore and Tristani (2013) (which considered the case of working-capital credit). Through the lenses of the current setup, the first result is explained by a strong balance-sheet channel (embedded in the first two models via a financial accelerator à la Kiyotaki and Moore (1997)), whereas the second by a strong input-price channel counteracting the balance-sheet one. In the current analysis both instances are possible in equilibrium depending on structural parameters.

Another distinguished feature of the current setup is that production and price decisions are taken jointly at the monopolistic firm level (as in the basic NK model). And this irrespectively of whether the firm is financially-constrained or not. This allows to study how monetary policy affects jointly firm-level output and prices. By contrast, with the exception of Gilchrist et al. (2017), in all other models production decisions are taken by perfectly competitive wholesalers, whereas pricing decisions are taken separately by monopolistic retailers which use the output of wholesalers as input. The setup in Gilchrist et al. (2017) is different than the current one because it looks at the particular case of customer-based markets instead of a standard monopolistic competition environment.

Finally, the paper contributes to the empirical literature on the effect of monetary policy on

[3] As argued by Jermann and Quadrini (2012), while the relevance of investment credit is obvious, working capital credit helps rationalizing the strong link between collateral and employment observed in the US data. As shown later in the analysis, it also allows to explain the estimated effect of monetary policy on constrained firms in the UK (in particular, the role of capital tangibility).

financially constrained versus unconstrained firms. Previous papers distinguished between the (average) effect of monetary policy on the two groups, using different proxies for financial constraints (e.g. young and not distributing dividends in Cloyne et al. (2018), high leverage in Ottonello and Winberry (2018), small in Gertler and Gilchrist (1994)). So far, I used the model as a guide to study in more detail the transmission of monetary policy to a subset of UK (public) firms which are likely to be credit-constrained, by extending the analysis in Cloyne et al. (2018). Going forward, I aim to further contribute to this literature by bringing supportive evidence for the new transmission channels of monetary policy identified in the theoretical analysis, namely for the "input price channel" and the "spillover channel".

# 3 Model

The analytical framework takes as a benchmark the Rotemberg version of the basic NK setup with working-capital paid in advance of sales[4]. This basic setup features three types of agents: a continuum of identical households which consume, work and save, differentiated monopolistic firms which produce, and a monetary authority which sets the one-period nominal interest rate. I add two types of financial frictions to this setup such that a set of firms end up credit-constrained in equilibrium. Without these frictions, the setup equals the basic model on all other dimensions[5].

I assume that there are two types of firms in the economy: firms which do not face any financing frictions ("unconstrained"), and firms which face two types of financial frictions ("constrained"). The first financial friction is a limit on their net worth. Specifically, these firms can finance with equity only a fraction of their desired physical capital (due to high issuance costs related to moral hazard for instance)[6]. For tractability, as in Gilchrist et al. (2017), constrained firms issue each period equity claims and do not retain profits[7]. The second financial friction is the requirement to secure nominal debt against collateral[8]. Because of these two frictions, the second type of firms

[4]The Calvo-version of the basic NK model (which is equal to a first order approximation to the Rotemberg version) is extensively described in Gali (2015) and Woodford (2003). In this baseline setup, working capital constraints do not affect equilibrium (Manea (2019)).

[5]Papers so far depart from the basic NK setup, not only by adding credit frictions, but also by assuming households and firms have distinct preferences. In Iacoviello (2005) and Andres et al. (2013) 'patient' households work, consume and save in equilibrium, whereas 'impatient' entrepreneurs produce, consume and borrow. In Carlstrom et al. (2010), households are risk-averse, whereas entrepreneurs are risk-neutral. Moreover, households supply two types of labor (one on which firms face credit collateral constraints).

[6]'Equity' should thus be understood more generally in the model as firm's *net worth*, namely as both fresh equity injections ('external equity') and retained earnings (internal equity).

[7]This allows to avoid technical difficulties due to heterogenous net worth (hence, output) of constrained firms.

[8]To microfound the latter friction, we may think of firms as being run by a special category of workers, 'managers',

face limits on how much they can produce. We can think of them as being relatively young, and thus without sufficient retained earnings so as to finance internally their operations, and without a well-established credit record so as to get external financing in an unconstrained manner. All constrained firms are identical and produce an identical amount. The same is true for unconstrained firms. I take the size of each set as exogenous. I note the size of the constrained group by $\phi$, and of the unconstrained one by $1-\phi$. Firm entry and exit flows are such that the distribution of the two types of firms is constant over time[9].

There are five markets in the model: goods, labor, capital, debt and equity. We may think of debt contracts as being intermediated via the banking sector. Thus, monetary authority's instrument is the interest rate on one-period nominal debt.

### 3.1 Households

The economy is populated by a continuum of identical infinitely-lived households of measure one. At each date, a representative household decides how much to consume ($C_t$), to work ($L_t$), and to invest in one-period nominal debt ($\mathscr{D}_t$) and equity shares ($\{\mathscr{E}_t(i)\}$) issued by firms, in order to maximise expected lifetime utility

$$E_0\Big\{\sum_{t=0}^{\infty}\beta^t Z_t\Big(\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{L_t^{1+\varphi}}{1+\varphi}\Big)\Big\}$$

subject to the sequence of budget constraints.

$$W_tL_t+(1+i_{t-1})\mathscr{D}_{t-1}+\int_0^1\mathscr{E}_{t-1}(i)\mathscr{R}^e_{t-1}(i)di\geq P_tC_t+\mathscr{D}_t+ \tag{1}$$

$$+\int_0^1\mathscr{E}_t(i)Q^e_t(i)di+T_t \tag{2}$$

and the solvency ('transversality') conditions

$$\lim_{T\to\infty}E_0\Big\{\beta^T\frac{U_{c,T}}{U_{c,t}}\frac{\mathscr{D}_T}{P_T}\Big\}\geq 0,\qquad \lim_{T\to\infty}E_0\Big\{\beta^T\frac{U_{c,T}}{U_{c,t}}\frac{\mathscr{E}_T(i)}{P_T}\Big\}\geq 0, \tag{3}$$

who can refuse to repay debt so as to maximize firms' shareholders' revenues. Creditors are thus only willing to lend against physical capital that can be seized and liquidated in case of repudiation.

[9] Namely, (i) at each date a mass of newborn firms enter the constrained set, while a mass of equal size simultaneously unexpectedly exit it to enter the (well-established) unconstrained set; (ii) simultaneously, a mass of equal size with the firms entering the unconstrained set unexpectedly exits the economy.

with $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ a standard Dixit-Stiglitz consumption index of differentiated goods with $\varepsilon$ a measure of substitutability among them and its unit price denoted by $P_t \equiv \int_0^1 \left(P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}$, $W_t$ the nominal wage, $i_t$ the one-period interest rate on nominal debt, $Q_t^e(i)$ the price of an equity claim in firm $i$, $\mathscr{E}_t(i)$ the number of equity claims in firm $i$, $T_t$ (lump-sum) monopolistic profits distributed by firms, and $Z_t$ an exogenous demand preference shifter described by $log(Z_t) = \rho_z log(Z_{t-1}) + \varepsilon_t^z$, $\varepsilon_t^z \sim \mathscr{N}(0, \sigma_z)$[10]. Let $\Lambda_{t,t+1} \equiv \beta\left(\frac{Z_{t+1}}{Z_t}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}$ denote the stochastic discount factor for one-period ahead (real) payoffs. Representative household's behavior is described by

$$C_t^{\sigma} L_t^{\varphi} = \frac{W_t}{P_t} \quad \forall t, \tag{4}$$

$$E_t\left\{\Lambda_{t,t+1}\Pi_{t+1}^{-1}\right\}(1+i_t) = 1 \quad \forall t, \tag{5}$$

$$E_t\left\{\Lambda_{t,t+1}\Pi_{t+1}^{-1}\mathscr{R}_t^e(i)\right\} = Q_t^s(i) \ \ \forall i \ \ \forall t, \tag{6}$$

alongside the budget constraints (1), and the transversality condition (3).

### 3.2 Firms

The model economy is populated by a continuum of firms in monopolistic competition which produce differentiated goods indexed by $i \in [0,1]$. At each date, firms have access to an identical constant returns to scale Cobb-Douglas technology

$$Y_t(i) = A_t K_t^{\alpha}(i) L_t^{1-\alpha}(i),$$

where $Y$ stands for output, $K$ for capital, and $log(A_t) = \rho_a log(A_{t-1}) + \varepsilon_t^a$, $\varepsilon_t^a \sim \mathscr{N}(0, \sigma_a)$ is a common stochastic productivity process. Capital is in fixed aggregate supply.

Firm $i$ enters period $t$ with predetermined capital $K_t(i)$ chosen at the end of $t-1$, and purchased at $t-1$ on a perfectly competitive market at price $Q_{t-1}^k$. Firms refinance each period their entire capital. The fraction $\theta(i)$ of $K_t(i)$ was financed by one-period equity claims at time $t-1$. Firm $i$ issued at that time $\mathscr{E}_t(i)$ claims equal to the number of capital units financed by equity, and priced each claim at the price of a capital unit, namely $Q_{t-1}^e = Q_{t-1}^k$[11], while it financed the rest

[10] As households are the patent owners of firm technology, they earn monopolistic profits.
[11] The same approached is followed in Gertler and Karadi (2011).

$(1-\theta(i))Q^k_{t-1}K_t(i)$ by nominal debt $\mathscr{D}_{t-1}(i)$ at interest rate $i_{t-1}$[12].

Firms need to pay the wage-bill in advance of sales ("cash-flow mismatch"). Subsequently, they need to finance each period not only physical capital, but also working-capital in advance of sales[13]. As in Jermann and Quadrini (2012) or Carlstrom et al. (2010)), they do so by issuing intratemporal (interest-free) debt. Firms can only issue debt against collateral. Firm's $i$ total debt cannot exceed a pledgeable fraction $\nu_t$ of its capital holdings (that can be seized in case of repudiation) at the end of the period (when debt is repaid):

$$W_tL_t(i) + (1+i_{t-1})\mathscr{D}_{t-1}(i) \leq \nu_t Q^k_t K_t(i), \quad log(\nu_t) = \rho_\nu log(\nu_{t-1}) + \varepsilon^\nu_t \text{[14]} \tag{7}$$

The sequence of events is as follows: at the beginning of $t$, subject to the credit collateral constraint (7), firm $i$ decides how much to produce $Y_t(i)$, hires workers $L_t(i)$ and sets the price $P_t(i)$ in the presence of Rotemberg (1982)-style adjustment costs $\zeta_t(\cdot) \equiv \frac{\xi}{2}Y_t\Big(\frac{P_t(i)}{P_{t-1}(i)} - 1\Big)^2 \geq 0$[15]. Once it receives its sales income at the end of the period, it resells capital, repays debt and equity returns, and redistributes monopolistic profits to households. After honoring all liabilities related to current production, it chooses capital for next period $K_{t+1}(i)$, and the production cycle starts over again.

Formally, at date $t$, firm $i$ takes as given its capital $K_t(i)$, its equity financing constraint $\theta(i)$, the wage $W_t$, the price level $P_t$, aggregate demand $Y_t$, and chooses $\{P_t(i), Y_t(i), L_t(i), K_{t+1}(i), \mathscr{D}_t(i)\}_{t\geq 0}$ to maximize its expected intertemporal profits:

$$E_0\Bigg\{\sum_{t=0}^{\infty}\Lambda_{0,t}\Bigg[\frac{P_t(i)}{P_t}Y_t(i) - \frac{W_t}{P_t}L_t(i) - \frac{\xi}{2}Y_t\Big(\frac{P_t(i)}{P_{t-1}(i)} - 1\Big)^2 + \frac{Q^k_t}{P_t}K_t(i) + \frac{\mathscr{D}_t(i)}{P_t}$$
$$- \Big(1+i_{t-1}\Big)\Pi_t^{-1}\frac{\mathscr{D}_{t-1}(i)}{P_{t-1}} - R^e_{t-1}\frac{\mathscr{E}_{t-1}(i)}{P_t} - \frac{Q^k_t}{P_t}K_{t+1}(i)\Bigg]\Bigg\},$$

subject to the sequence of collateral constraints:

$$\lambda^1_t(i) : \nu_t\frac{Q^k_t}{P_t}K_t(i) - (1+i_{t-1})\frac{\mathscr{D}_{t-1}(i)}{P_{t-1}}\Pi_t^{-1} \geq \frac{W_t}{P_t}L_t(i) \;\; \forall t, \tag{8}$$

[12] As in Iacoviello (2005), nominal debt (as opposed to inflation-indexed one) is justified by the observation that in low inflation countries almost all debt contracts are in nominal terms.

[13] As working capital loan is intra-temporal (as in Jermann and Quadrini (2012) or Carlstrom et al. (2010)), the effects of their interest rate on the credit constraint tightness are ignored.

[14] If firms repudiate their debt obligations, banks can reposes their collateral assets only by paying a proportional transaction cost (e.g. Quadrini (2011)), so $\nu_t < 1$.

[15] I chose Rotemberg (1982)-style nominal price rigidities because they allow price and quantity decisions to be jointly taken by standard monopolistic firms in the presence of collateral constraints (Jermann and Quadrini (2012)).

demand constraints:

$$\lambda_t^2(i) : \Big(\frac{P_t(i)}{P_t}\Big)^{-\varepsilon} Y_t - Y_t(i) = 0 \;\; \forall t, \tag{9}$$

technology constraints:

$$\lambda_t^3(i) : Y_t(i) - A_t K_t^{\alpha}(i) L_t^{1-\alpha}(i) = 0 \;\; \forall t, \tag{10}$$

and budget constraints[16]

$$\begin{aligned} &\Big[\frac{P_t(i)}{P_t} Y_t(i) - \frac{W_t}{P_t} L_t(i) - \frac{\xi}{2} Y_t \Big(\frac{P_t(i)}{P_{t-1}(i)} - 1\Big)^2 + \frac{Q_t^k}{P_t} K_t(i) - \Big(1 + i_{t-1}\Big) \frac{\mathscr{D}_{t-1}(i)}{P_t} \\ &- \frac{R_{t-1}^e(i)\mathscr{E}_{t-1}(i)}{P_t} - \frac{T_t}{P_t}\Big] + \frac{\mathscr{D}_t(i)}{P_t} + \frac{Q_t^e(i)\mathscr{E}_t(i)}{P_t} = \frac{Q_t^k}{P_t} K_{t+1}(i) \;\; \forall t. \end{aligned} \tag{11}$$

where $\lambda_t^1(i)$, $\lambda_t^2(i)$ and $\lambda_t^3(i)$ are the lagrange multipliers associated to each of the three binding constraints in firm (i)'s maximization problem. Since firm $i$ fully repays gross equity returns and monopolistic profits at the end of each period (the term in brackets is zero), and equity issued at $t$ equals $\theta(i)Q_t^k K_{t+1}(i)$, firm $i$'s budget constraint (11) implies:

$$\mathscr{D}_t(i) = (1 - \theta(i)) Q_t^k K_{t+1}(i) \tag{12}$$

We can thus replace the expression of $\mathscr{D}_t(i)$ from (12) in firm $i$'s optimization problem, and eliminate budget constraint (11) alltogether. The Lagrangian method gives the following optimality conditions for firm $i$'s behaviour:

$$\xi \Pi_t(i)\Big(\Pi_t(i) - 1\Big)\frac{Y_t}{Y_t(i)} = \xi E_t\Big\{\Lambda_{t,t+1}\Big[\frac{Y_{t+1}}{Y_t(i)}\Pi_{t+1}(i)\Big(\Pi_{t+1}(i) - 1\Big)\Big]\Big\} + \Big(\frac{P_t(i)}{P_t} - \varepsilon\lambda_t^2(i)\Big),$$

$$\frac{P_t(i)}{P_t} - \lambda_t^2(i) - MC_t(i)\Big(1 + \lambda_t^1(i)\Big) = 0,$$

$$\begin{aligned} \frac{Q_t^k}{P_t} =& E_t\Big\{\Lambda_{t,t+1}\Big[\frac{Q_{t+1}^k}{P_{t+1}} + \Big(\frac{P_{t+1}(i)}{P_{t+1}} - \lambda_{t+1}^2(i)\Big)\alpha\frac{Y_{t+1}(i)}{K_{t+1}(i)} + \\ &+ \lambda_{t+1}^1(i)\Big(\nu_{t+1}\frac{Q_{t+1}^k}{P_{t+1}} - (1 + i_t)(1 - \theta(i))\frac{Q_t^k}{P_t}\Pi_{t+1}^{-1}\Big)\Big]\Big\}, \end{aligned}$$

$$\begin{aligned} \lambda_t^1(i)\Big(\nu_t\frac{Q_t^k}{P_t}K_t(i) - \Big(1 + i_{t-1}\Big)\frac{\mathscr{D}_{t-1}(i)}{P_{t-1}}\Pi_t^{-1} - \frac{W_t}{P_t}L_t(i)\Big) = 0, \;\; \lambda_t^1(i) \geq 0 \\ \nu_t\frac{Q_t^k}{P_t}K_t(i) - \Big(1 + i_{t-1}\Big)\frac{\mathscr{D}_{t-1}(i)}{P_{t-1}}\Pi_t^{-1} - \frac{W_t}{P_t}L_t(i) \geq 0, \;\; \forall t, \end{aligned}$$

alongside the sequence of demand (9) and technological (10) constraints. $MC_t(i) \equiv \frac{1}{1-\alpha}\frac{W_t}{P_t}\frac{L_t(i)}{Y_t(i)}$

[16] According to the budget constraint, capital is financed with new debt, new equity and previous capital returns net of old debt, equity returns and redistributed monopolistic profits.

denotes firms $i$'s real marginal cost.

The model economy is populated by two sets of firms: a set of mass $\phi$ denoted by $\Theta^c$ with $\theta(i)$ low enough for collateral constraints to bind in the vicinity of steady-state, and another set denoted by $\Theta^u$ of mass $1-\phi$ with $\theta(i)=1$. Firms within each set behave identically. Variables related to the constrained set are indexed by $'c'$, and the ones to the unconstrained one by $'u'$.

### 3.3 Monetary authority

Unless otherwise stated, the monetary authority sets the nominal interest rate

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_y \widehat{y}_t + \varepsilon_t^m, \tag{13}$$

where $\pi_t \equiv log(\Pi_t)$, $\widehat{y}_t \equiv log(Y_t) - log(Y)$ and $\varepsilon_t^m = \rho_m \varepsilon_{t-1}^m + \epsilon_t^m$, $\epsilon_t^m \sim \mathcal{N}(0, \sigma_m)$.

### 3.4 Market clearing

Market clearing is imposed:

- for each good variety $i$: $Y_t(i) = C_t(i) + \xi_t(i)$, $\forall i$ with $\xi_t(i)$ the price adjustment costs in terms of variety $i$[17];

- on labor market where labor supplied by households must equate labor demanded by firms: $L_t = \int_0^1 L_t(i)di = \phi L_t^c + (1-\phi)L_t^u$;

- on capital market where firms' aggregate demand must equate the (exogenously fixed) aggregate supply: $\bar{K} = \int_0^1 K_{t+1}(i)di = \phi K_{t+1}^c + (1-\phi)K_{t+1}^u$;

- on debt market where demand equals supply subject to collateral requirements, for both inter-temporal debt $\mathscr{D}_t = \int_0^\phi (1-\theta^c) Q_t^k K_{t+1}(i)di = \phi(1-\theta^c)Q_t^k K_{t+1}^c$, and intra-temporal one $\mathscr{D}_t^i = W_t L_t$;

- on equity market where the value of equity claims demanded by households must equate the value issued by constrained and unconstrained firms[18]:

$$\int_0^1 \mathscr{E}_t(i) Q_t^e(i) di = \int_0^\phi Q_t^e(i)\mathscr{E}_t(i)di + \int_\phi^1 Q_t^e(i)\mathscr{E}_t(i)di \tag{14}$$

$$= \phi\theta^c Q_t^k K_{t+1}^c + (1-\phi) Q_t^k K_{t+1}^u \tag{15}$$

[17]The allocation of adjustment costs among varieties is the same as for consumption.
[18]Households are the ultimate owners of all firms in the economy.

# 4 Monetary policy transmission

I split the theoretical analysis of the transmission of monetary policy in the presence of constrained and unconstrained firms in three parts. In the first part, I investigate how the financial constraints faced by firms shape the mechanisms of transmission of monetary policy relatively to the credit-frictionless benchmark in Gal015) or Woodford (2003). In the second part, I analyze how the effect of monetary policy at the firm level depends on structural parameters, focusing on examples that can be tested empirically. In the third stage, I study how the reaction of *macroeconomic* variables is affected by the share of constrained firms, and how results hinge on structural parameters. I base my theoretical analysis on a first order log-linear approximation of the model in the vicinity of the non-stochastic zero-inflation steady-state[19]. Notation is standard: small caps stand for log-levels, ($\widehat{\ }$) for log-deviation from steady-state, while the absence of a time subscript denotes a steady-state value.

## 4.1 Transmission mechanisms

Let's consider first the limiting case of $\phi = 0$. Given nominal rigidities  Rotemberg, all firms choose the same price and output levels. Equilibrium dynamics may be summarized by the following three equations:

$$\widehat{y}_t = E_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\big(\widehat{i}_t - E_t\{\pi_{t+1}\}\big) + (1-\rho_z)z_t \tag{16}$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda\Big(\sigma + \frac{\alpha+\varphi}{1-\alpha}\Big)\widehat{y}_t - \lambda\frac{1+\varphi}{1-\alpha}a_t \tag{17}$$

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_y \widehat{y}_t + \varepsilon_t^m \tag{18}$$

For $\xi$ calibrated such that $\lambda$ equals the value in the Calvo version, the model is isomorphic (up to a first order) to the basic NK setup in Gali (2015, Chapter 3).

A monetary tightening reduces aggregate demand and is transmitted to firms' decisions through associated declines in their individual demand schedules[20]. In the credit-frictionless environment, firms can produce as much as they want subject to their demand and price setting constraints.

[19] Shocks are small enough for supply to be non-rationed and credit constraints to bind.

[20] Output and inflation dynamics are independent of asset prices in the credit-frictionless limit. For a detailed analysis of monetary policy transmission and optimal design in this case see for instance Gali (2015), Chapters 3 to 5.

Figure 2 depicts the response of a firm to a monetary tightening in this limiting case. Specifically, it shows how as the negative monetary impulse reduces its demand schedule, namely it shifts it to the left from $D_0$ (black solid line) to $D_1$ (navy dotted line), the firm simultaneously readjusts downwards its output (from $Q_0$ to $Q_1$) and price (from $P_0$ to $P_1$) given the marginal cost schedule (Cmg line) and price adjustment costs[21]. In this environment, the (real) marginal cost schedule of the economy is also affected in equilibrium. However, its endogenous variation is never strong enough so as to overturn at the firm-level the sign of the direct effect of the monetary impulse via the demand channel. This is why, the "input-price channel" does not play a key independent role in the transmission of monetary policy when the economy is populated only by unconstrained firms.

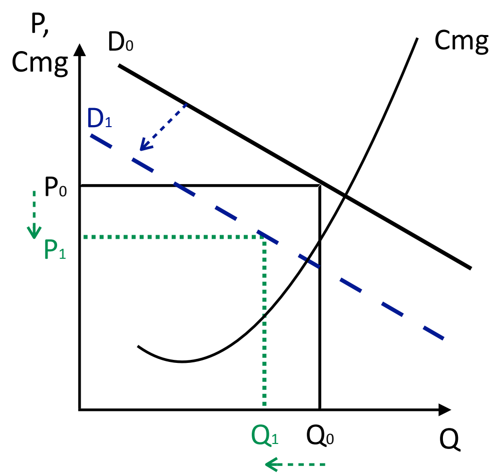



Figure 2: Firm response to a monetary tightening in the credit-frictionless limit

In the general case when some of the firms are constrained, namely $\phi \in (0, 1)$, monetary impulses are still directly exerted via the demand/saving channel. This is because the structure of the demand-side of the model economy remains the same as the one in the credit-frictionless limit. A series of indirect effects acquire however new roles in the transmission of monetary policy at the firm level. Moreover, monetary policy transmits to constrained firms through different mechanisms than to unconstrained ones, and spillover effects between the two types firms play a key role in the transmission of monetary policy.

At each date, the output of *a constrained firm* is determined by its collateral constraint (30). This

[21] Firm's real marginal cost schedule also shifts in equilibrium, but it is held fixed in figure 2 for ease of exposition. Price adjustment costs (not modeled) imply that the firm takes into account as well future marginal cost and demand schedules.

constraint implies that given outstanding capital $\widehat{k}_t^c$ and nominal debt (and exogenous disturbances), the production of such a firm depends on (i) the current price of capital pledged as collateral $\widehat{\varrho}_t^k$, (ii) price inflation $\pi_t$, and (iii) the prices of the inputs financed against collateral (here, real wage $\widehat{\omega}_t$):

$$\begin{aligned}\widehat{\boldsymbol{y}}_{\boldsymbol{t}}^{\boldsymbol{c}} = \widehat{k}_t^c - \frac{(1-\alpha)\beta^{-1}(1-\theta^c)}{\nu-\beta^{-1}(1-\theta^c)}\left[\widehat{\varrho}_{t-1}^k + \left(\widehat{i}_{t-1}-\pi_t\right)\right] \\ + \frac{(1-\alpha)\nu}{\nu-\beta^{-1}(1-\theta^c)}\left(\widehat{\varrho}_t^k-\nu_t\right) - (1-\alpha)\,\widehat{\omega}_t + a_t\end{aligned}$$

Thus, a transitory monetary impulse transmits to the current production of a constrained firm via its indirect effects on (i) the real price of capital pledged as collateral, (ii) the real value of outstanding (nominal) debt (via inflation), and (iii) the real prices of the inputs financed against collateral:

$$\frac{\partial \widehat{y}_t^c}{\partial \varepsilon_t^m} = \overbrace{\frac{(1-\alpha)\nu}{\nu-\beta^{-1}(1-\theta^c)}\frac{\partial \widehat{\varrho}_t^k}{\partial \varepsilon_t^m}}^{\text{balance-sheet}} + \overbrace{\frac{(1-\alpha)\beta^{-1}(1-\theta^c)}{\nu-\beta^{-1}(1-\theta^c)}\frac{\partial \pi_t}{\partial \varepsilon_t^m}}^{\text{nominal-debt}} \overbrace{-(1-\alpha)\frac{\partial \widehat{\omega}_t}{\partial \varepsilon_t^m}}^{\text{input-price}}$$

The first mechanism is a facet of the standard "balance-sheet channel" which refers to the fact that a rise in the policy rate depresses asset prices, and hence shrinks the value of firm's collateral[22]. The second one is the "nominal debt-channel": since (outstanding) debt is nominal, monetary policy affects its real value via inflation[23]. Both mechanisms have been already discussed in the literature (e.g. the first one in Bernanke and Gertler (1995), both in Iacoviello (2005)). The third one however, working via the real prices of the inputs financed against collateral, to the best of my knowledge, has been overlooked so far. Its key relevance in this context is given by the consideration of working-capital credit secured against collateral. Firm heterogeneity plays also an important role in determining its strength in equilibrium. I call this new channel the *input-price channel*.

Importantly, these three transmission mechanisms affect production by constrained firms in opposite directions. For instance, a transitory monetary tightening simultaneously *lowers* the production possibilities of these firms by pushing downwards the price of their pledgeable collateral via the "balance-sheet" mechanism, and *expands* them by depressing real input prices (via the

[22]Bernanke and Gertler (1995) note that 'many observers would agree that the crash of Japanese land in the latter 1980s was the result (at least in part) of monetary tightening and that this collapse in asset values reduced the credit-worthiness of many Japanese corporations, contributing to the ensuing recession.' They also mention that according to Borio et al. (1994) 'a similar pattern of asset price boom and bust leading to real fluctuations occurred during the 1980's in a number of major industrialized countries'.

[23]The nominal debt channel plays an important role in the 'debt-deflation' theory of the (1929- 1939) 'Great Depression in US' proposed by Fisher (1933).

decline in output, hence, in input demand). Depending on how inflation reacts in equilibrium, "debt-deflation" might further affect them in either direction. Thus, *a priori*, monetary policy may steer current output of constrained firms in either direction depending on the relative strength of each mechanism (and hence, on structural parameters).

These three transmission mechanisms are further relevant for the pricing decisions of constrained firms. This is because their prices depend, alongside aggregate demand ($y_t$, $p_t$), on the (constrained) level of output ($y_t^c$):

$$p_t^c = \frac{1}{\varepsilon}(y_t - y_t^c) + p_t \tag{19}$$

Subsequently, a monetary tightening, which pushes output of constrained firms downwards via the balance-sheet channel, and upwards via the input-price channel, has a simultaneous opposite effect on their prices via these channels (note that $y_t^c$ and $p_t^c$ are inversely related in the downward sloping demand curve (19)).

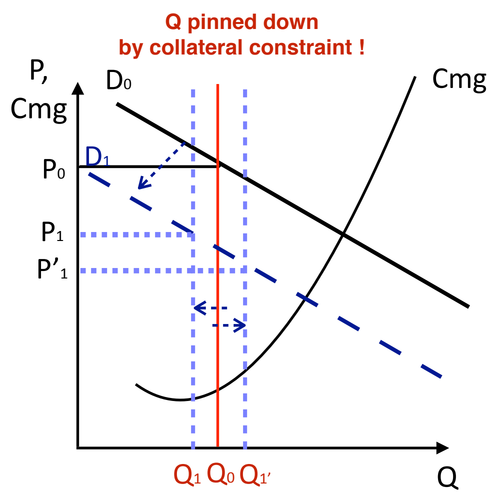



Figure 3: Transmission of a monetary tightening to a constrained firm

Finally, as shown by equation (20), monetary policy further affects future production levels of constrained firms via their investment decision ("investment channel") and via the associated debt level:

$$\frac{\partial E_t\{\widehat{y}_{t+1}^c\}}{\partial \varepsilon_t^m} = \overbrace{\frac{\partial \widehat{k}_{t+1}^c}{\partial \varepsilon_t^m}}^{\text{investment channel}} \quad \overbrace{- \frac{(1-\alpha)\beta^{-1}(1-\theta^c)}{\nu - \beta^{-1}(1-\theta^c)} \Big( \frac{\partial \widehat{\varrho}_t^k}{\partial \varepsilon_t^m} + \frac{\partial \widehat{i}_t}{\partial \varepsilon_t^m} \Big)}^{\text{debt overhang}} \tag{20}$$

The sign and strength of these effects are directly linked to the ones on current output.

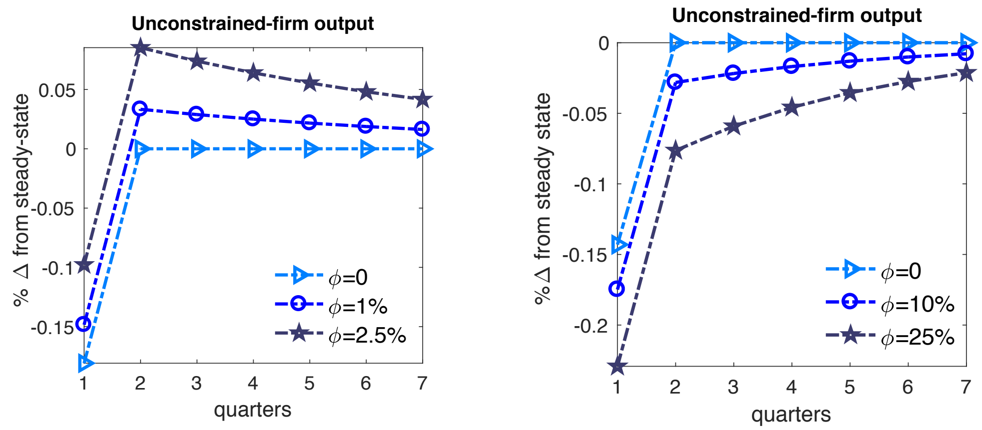
Figure 3 summarizes the transmission of a transitory monetary tightening to a constrained firm. On the one hand, it shows that the monetary impulse affects its output ($Q_0$) via the credit constraint and that the equilibrium effect can be of either sign, namely the vertical red line may shift to either of the two purple lines (i.e. to either $Q_1$ or $Q_1'$). On the other hand, it shows that monetary policy affects the price of a constrained firm (from $P_0$ to $P_1$ or $P_1'$) via these effects and the shift engineered in its demand schedule (from $D_0$ to $D_1$).

Now, after having analyzed how monetary policy transmits to constrained firms, let's turn to how it transmits to *unconstrained ones*. On the one hand, monetary policy affects their output $y_t^u$ and pricing decisions $p_t^u$ by shifting their demand schedule:

$$y_t^u = -\varepsilon p_t^u + (\varepsilon p_t + y_t) \tag{21}$$

The demand for the goods of a monopolistic firm depends on the prices set by its competitions. Thus, in this heterogenous firm environment, the demand faced by unconstrained firms depends on the prices set by constrained firms.

Subsequently, monetary policy shifts the demand schedule of an unconstrained firm both *directly* via the consumption/saving decision of the household, and *indirectly* via its effects on the pricing (and production) decisions of constrained firms. Thus, when monetary policy pushes upwards or downwards the output of constrained firms, it simultaneously pushes the output and prices of unconstrained firms in the opposite direction. Furthermore, the choices made by unconstrained firms depend on the (equilibrium) effect of monetary policy on their marginal costs. These equilibrium effects depend also in the current heterogenous firm setup on the effects of monetary policy on constrained firms.




(a) Dominant balance-sheet channel

(b) Dominant input-price channel

Figure 4: Transmission spillovers and the response of an unconstrained firm to a monetary tightening

To get an intuition on how transmission spillovers from constrained firms affect the decisions of unconstrained firms, figure 4 shows the response of unconstrained firms to a rise in the policy rate as the share of constrained firms increases. It considers separately the case where the balance-sheet channel dominates (left panel), and the case where the input-price channel dominates (right panel). As previously shown, in response to a monetary tightening, a strong balance-sheet channel translates in (i) strong positive pressures on the prices of constrained firms, and (ii) strong negative pressures on input prices, and hence on marginal costs. Both these two indirect effects counteract the negative effects on unconstrained firms via the demand channel. As the share of constrained firms increases, these (counteracting) spillovers become stronger. Consistently, the left panel in figure 4 shows how the latter dampen the net equilibrium effect of monetary policy on unconstrained firms, and how, once strong enough, they switch the sign of the net effect in their direction. As shown in the right panel of figure 4, the opposite is true in the case of a dominant input-price channel, where the spillovers enhance instead the negative effect on unconstrained firms via the demand channel.

## 4.2 Firm-level transmission and structural parameters

We have seen that monetary policy has a differential effect on constrained and unconstrained firms. But how does this difference depend on the features of the economy? I now use a calibrated version of the model to show that the differential response of constrained and unconstrained firms increases in the tightness of financial constraints and in the level of nominal input price stickiness.

In the baseline calibration (table 1) non-financial parameters equal the textbook values in Gali

(2015), the capital pledgeability ratio $\nu$ takes a value similar to the one in Iacoviello (2005), the fraction of constrained firms $\phi$ is set to its estimate in the UK data, and $\theta^c$ is such that around 20% of capital in the constrained set is financed by net worth. A time period in the model is one quarter. In all experiments, for ease of comparison with estimated dynamic responses, I consider the effect of a transitory monetary tightening of 25 basis points (figure 5).

Table 1: Baseline calibration

| | Parameter | Value |
|---|---|---|
| Non-financial parameters | | |
| Intertemporal elasticity of substitution | $\sigma$ | 1 |
| Discount factor | $\beta$ | 0.99 |
| Inverse Frisch elasticity of labor supply | $\varphi$ | 5 |
| Share of capital in total output | $\alpha$ | 0.25 |
| Output variety elasticity of substitution | $\varepsilon$ | 9 |
| Price adjustment cost parameter | $\xi$ | average $\kappa$ as with Calvo for $\theta = 0.75$ |
| Inflation coefficient Taylor rule | $\phi_\pi$ | 1.5 |
| Output coefficient Taylor rule | $\phi_y$ | 0.5/4 |
| | | |
| Credit frictions | | |
| Share of constrained firms | $\phi$ | 0.2 |
| Capital pledgeability ratio as collateral | $\nu$ | 0.8 |
| Fraction of capital financed by net worth | $\theta^c$ | 0.23 |

I start studying how the tightness of credit constraints affects the transmission of monetary policy by looking at the effect of the capital pledgeability ratio $\nu$. Figure 6 shows that the lower this ratio, the stronger the reduction in output, investment and working capital by constrained firms in response to a rise in the policy rate (navy lines in left panels *versus* purple lines in right panels). Otherwise stated, a lower pledgeability of capital implies a relatively weaker input price channel[24].

[24] As opposed to the current the model, the model in Iacoviello (2005) predicts a positive relation between capital tangibility and the effect of monetary policy. The difference comes from the consideration of working capital credit in the current analysis, alongside investment credit (Iacoviello (2005) considers only investment credit).

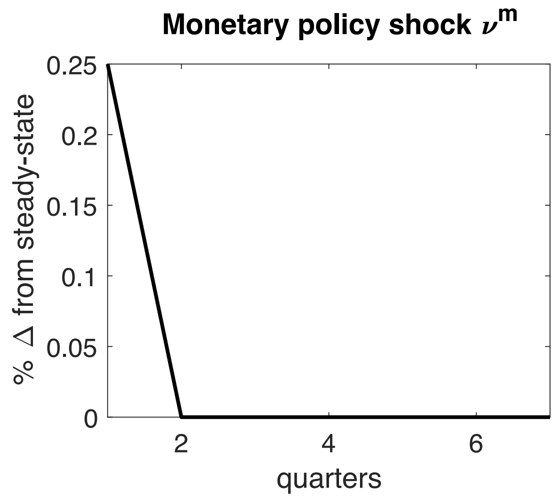
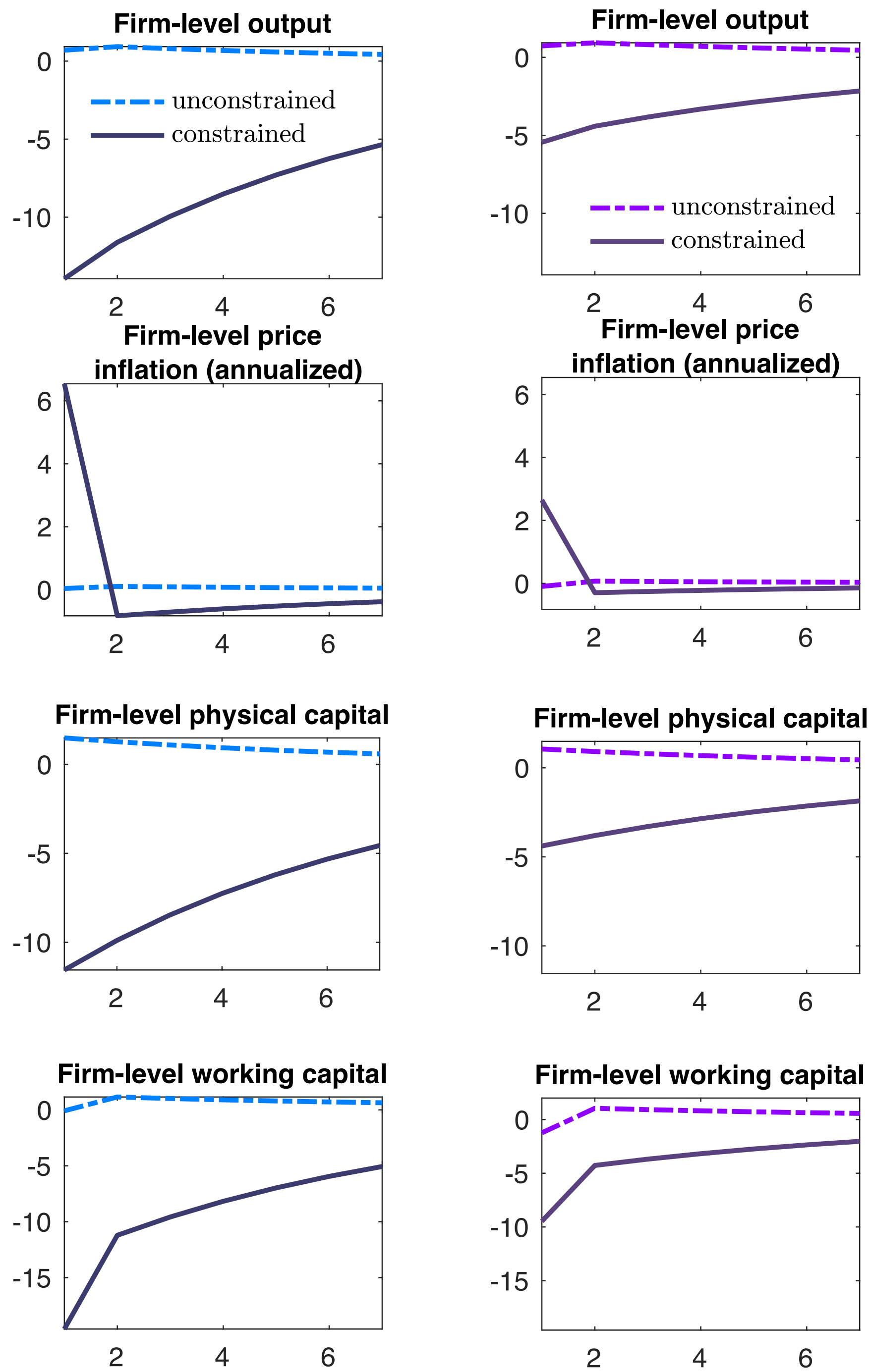



Figure 5: A transitory monetary tightening (25bp)
Note: Y-axis: % deviation from steady-state. X-axis: quarters

These results are explained by a decrease in the share of working capital credit to total credit as capital pledgeability $\nu$ decreases, as shown by its steady-state expression $1 - \frac{\beta^{-1}(1-\theta^c)}{\nu}$[25]. Thus, as capital becomes less plegeable, at given prices of labor and capital, the input-price channel gets relatively weaker, while the balance-sheet one gets relatively stronger. This is because constrained firms can finance to a lesser extent working capital credit against physical capital. Results are robust to changes in the share of constrained firms in the economy, and, as shown in figure 27 in the Appendix, also to an alternative calibration with a lower contribution of physical capital as production input as in Iacoviello (2005).

Furthermore, consistent with a stronger balance-sheet channel, given the downward-sloping demand schedule in equation (19), prices of constrained firms are steered more strongly upwards when capital is less pledgeable. This is because the stronger negative effect on constrained' firms output via the balance-sheet channel translates in a stronger positive effect on their prices. Also consistently, when capital pledgeability is low, transmission spillovers counteract to a larger extent the direct (negative) effects of a monetary tightening on unconstrained firms via the demand channel. Differences under baseline calibration are however small, and can be most easily noticed in the case of physical capital and working-capital.

[25]This ratio is computed using the expression of the collateral constraint (47) on page 62.




(a) Low capital pledgeability (b) High capital pledgeability

Figure 6: Dynamic responses to a transitory 25 basis points monetary tightening
Note: Y-axis: % deviation from steady-state. X-axis: quarters

Similar results are obtained when we consider the *net worth of constrained firms* (figure 28 in the Appendix). Specifically, firms with a lower net worth (and, hence, tighter credit constraints) reduce more strongly production and increase more their prices in response to a monetary tightening. Moreover, transmission spillovers counteract to a larger extent the negative effect of the monetary tightening on unconstrained firms via the demand channel. As shown by the steady-state share of working capital credit to total credit $1-\frac{\beta^{-1}(1-\theta^{c})}{\nu}$, these dynamics are also explained by a decrease in the ratio of working capital credit in total credit as net worth $\theta^{c}$ decreases.

Again same conclusions obtain under an alternative calibration with a lower contribution of physical capital as production input as in Iacoviello (2005) (figure 29 in the Appenidx). Interestingly also, under this alternative calibration, for high enough net worth, the relative strength of the input-price channel increases to the extent that it surpasses the one of the balance-sheet channel. Subsequently, output of constrained firms may be steered in a positive unconventional direction by a monetary tightening (solid navy line in figure 7). So, under the alternative calibration with $\alpha = 0.03$, there is a non-monotonic relation between firm new worth and the firm-level output response to monetary policy. Specifically, when the net worth of a firm is low, the firm is constrained and responds strongly to monetary policy. For higher levels of net worth, but low enough for the firm to remain constrained, the firm starts responding less and less to monetary policy, up to a point when it starts responding in an unconventional direction (region 1 in figure 8). This is because, as firm's net worth increases, the relatively strength of the input-price channel is enhanced, and ultimately it becomes stronger than the balance-sheet one. However, as its net worth increases and surpasses the level over which the firm becomes unconstrained, its output responds again in the conventional direction (region 2 in figure 8).

Finally, note in the right panel of the first row in figure 7 how in the case of a dominant input-price channel, constrained firms decrease their prices to accommodate the increase in output. Thus, in this particular case, spillovers push upwards the marginal costs of unconstrained firms and downwards their demand schedules, and hence reinforce the negative effects of the monetary tightening on unconstrained firms via the demand channel.

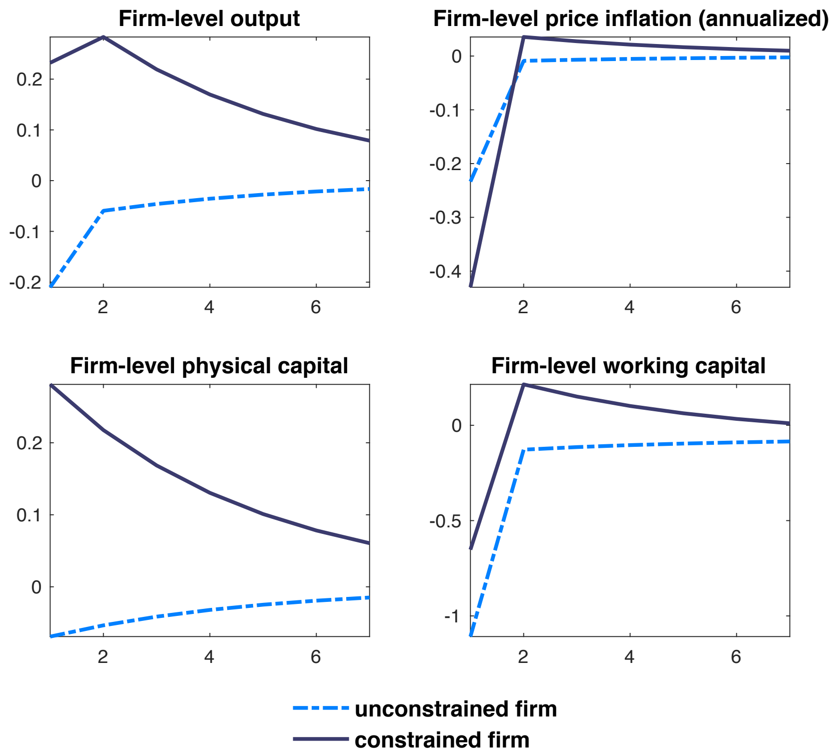



Figure 7: Dynamic response to a transitory monetary tightening (case with a relatively stronger input price channel: alternative calibration with $\alpha = 0.03$ as in Iacoviello (2005) and lower leverage $\theta^c = 0.45$)
Note: Y-axis: % deviation from steady-state. X-axis: quarters

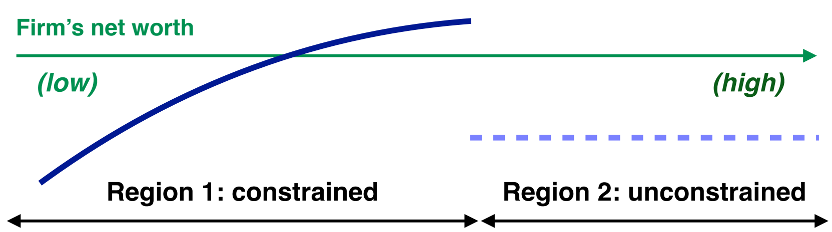



Figure 8: A firm's output response as a function of its net worth under the alternative calibration with $\alpha = 0.03$: navy solid line shows its response for values of net worth low enough for the firm to be constrained; light-blue dotted line shows its response for values of net worth high enough for the firm to be unconstrained

So far wages were assumed to be flexible. I now look at the effect of *wage stickiness* as a proxy (more generally) for input price stickiness. The sluggish adjustment of input prices is expected to weaken the relative strength of the input-price channel in equilibrium. To introduce wage stickiness, I follow the approach in Erceg, Henderson and Levin (2000). Specifically, I assume that the model economy is populated by a large number of identical households, each made up of a continuum of members specialized in a different labor service $j \in [0,1]$. Household labor is now defined by an index of labor types:

$$L_t \equiv \int_0^1 \frac{L_t(j)^{1+\varphi}}{1+\varphi} dj \tag{22}$$

Labor decisions for each type $j$ are taken at a union level with monopoly power over that labor type. Income is pooled within each household. The optimization problem of a typical household is identical to the one with flexible wages described in section 3.1, with the exception that $L_t$ is now taken as given. Firms use all labor types and $L_t$ represents the optimal mixture of these types. Otherwise, firms behave identically as in the flexible wage case[26].

In the presence of nominal wage rigidities, aggregate wage dynamics are described up to a first order approximation (in logs) by

$$\pi_t^w \approx \beta E_t\{\pi_{t+1}^w\} - \lambda_w \widehat{\mu}_t^w \tag{23}$$

where $\pi_t^w \equiv log\Big(\frac{W_t}{W_{t-1}}\Big)$ and $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w = \Big(\omega_t - mrs_t\Big) - \mu^w$ with $mrs_t \equiv \sigma c_t + \varphi l_t$ the economy's average marginal rate of substitution. Imperfect adjustment of nominal wages precludes real wages from moving one-for-one with households' average marginal rate of substitution. The wage inflation equation (23) replaces households' labor supply equation $\widehat{\omega}_t = \sigma\widehat{c}_t + \varphi\widehat{l_t}$ in the flexible wage case. Besides this modification, the model retains its baseline structure presented in section 3. To calibrate the new structural parameters, I follow Gali (2015), Chapter 6 and I set $\lambda_w$ to match an average duration of wage spells of four quarters and, respectively, an average unemployment rate of 5% in the credit frictionless limit. A table reviewing the complete calibration is included in the Appendix on page 60. The credit-frictionless benchmark $\phi = 0$ is isomorphic (up to a first order approximation) to the one described in Gali (2015), Chapter 6.

When both types of firms populate the model economy, the nature of transmission mechanisms at the firm level is similar to the one with flexible wages. The sluggishness of nominal wage adjustments

[26] For details see Gali (2015), Chapter 6.

however directly weakens the input-price channel, and reinforces the strength of the balance-sheet channel in equilibrium. As a result, under baseline calibration, activity of constrained firms declines more when wages are sticky (right panels figure 30 in the Appendix) compared to when they are flexible (left panels in figure 30). Consistently, both the positive pressures on their prices and the spillover effects counteracting the negative impact of the monetary tightening on unconstrained firms are stronger[27].

### 4.3 Aggregate effect of monetary policy

I looked so far at how monetary policy transmits at the firm-level in an environment where some of the firms are financially-constrained. I now analyze how credit frictions shape the aggregate response of the economy to monetary policy. I have three new main findings with respect to existing literature. (i) Credit frictions do not necessarily amplify the response of aggregate activity to monetary policy. (ii) Observing constrained firms responding more than unconstrained ones to monetary policy does not necessarily imply that the aggregate response is also amplified by credit frictions. (iii) "Price puzzles" may emerge because monetary policy simultaneously shifts both aggregate demand and supply schedules of the economy.

The first two results arise because of the opposing nature of both transmission channels to the output of constrained firms, and of spillovers between the two types of firms. Aggregate amplification occurs in the case of a dominant balance-sheet channel when monetary policy affects more constrained firms than the unconstrained, only if (counteracting) spillovers to the unconstrained are weak. As shown in figure 9, this is the case under baseline calibration. Specifically, in this case, constrained firms reduce more production and investment than unconstrained ones in response to a rise in the interest rate (bottom panels), and, also, as the share of constrained firms ($\phi$) increases, the effect of monetary policy on aggregate activity is amplified (top panels).

This is not however generally the case. Under the alternative calibration with $\alpha = 0.03$ as in Iacoviello (2005) (figure 10), because of general equilibrium (spillover) effects, even though constrained firms respond more than the unconstrained (bottom panels), aggregate responses are not (significantly) amplified as the share of constrained firms increases (top panels)[28]. One interesting

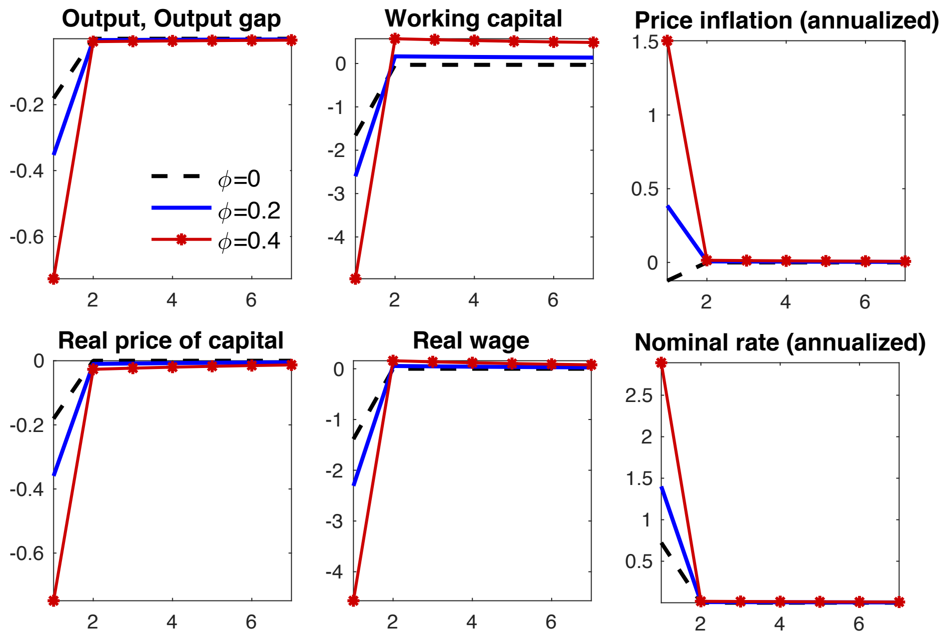
[27]Same qualitative results obtain for the alternative calibration of $\alpha = 0.03$ proposed by Iacoviello (2005) (figure 31), as well as for different shares of constrained firms.

[28]Moreover, note that for both such calibrations, the stronger response of investment by constrained firms is not associated to an amplification of aggregate investment since the latter is always equal to its credit frictionless

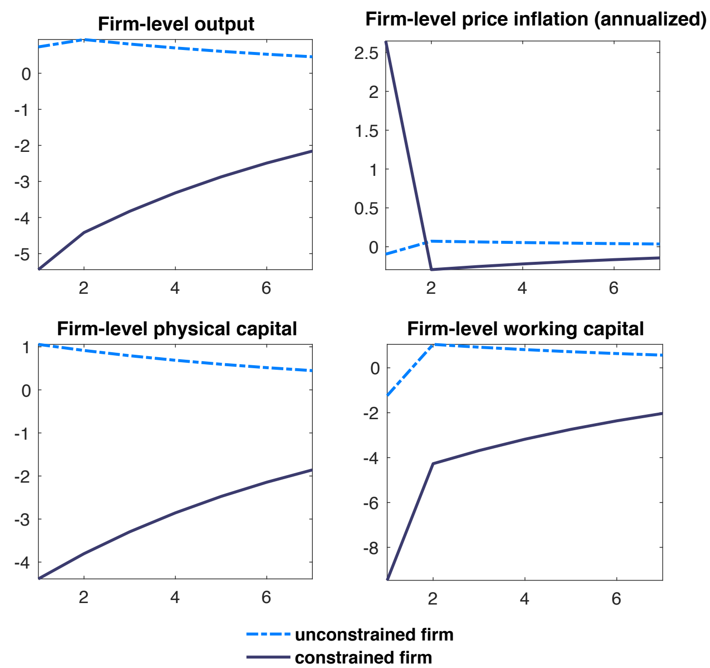
implication of these results is that observing constrained firms responding more (on average) than the unconstrained (as observed in countries such as the UK or the US), does not necessarily imply that credit frictions amplify the response of aggregate activity to monetary policy (as usually concluded by empirical studies so far).

Finally, figure 11 further shows an example where credit frictions actually dampen the response of aggregate activity to monetary policy, namely where, as the share of constrained firms in the economy increases, aggregate activity declines less in response to a rise in the interest rate. This case characterizes the one with a dominant input price channel depicted in figure 7, where output of constrained firms is steered in an unconventional positive direction by a monetary tightening.

counterpart.

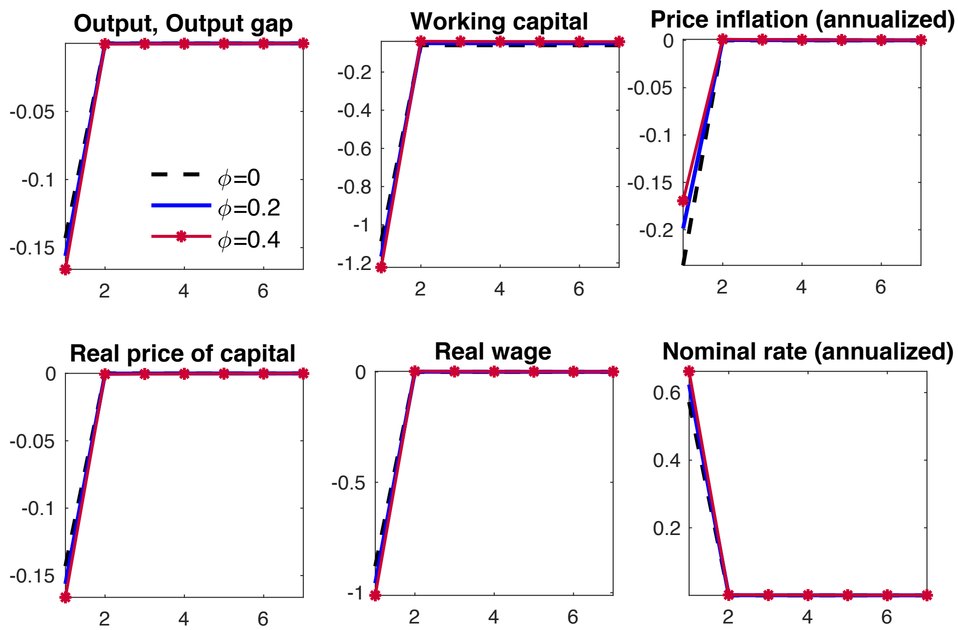



(a) Aggregate responses

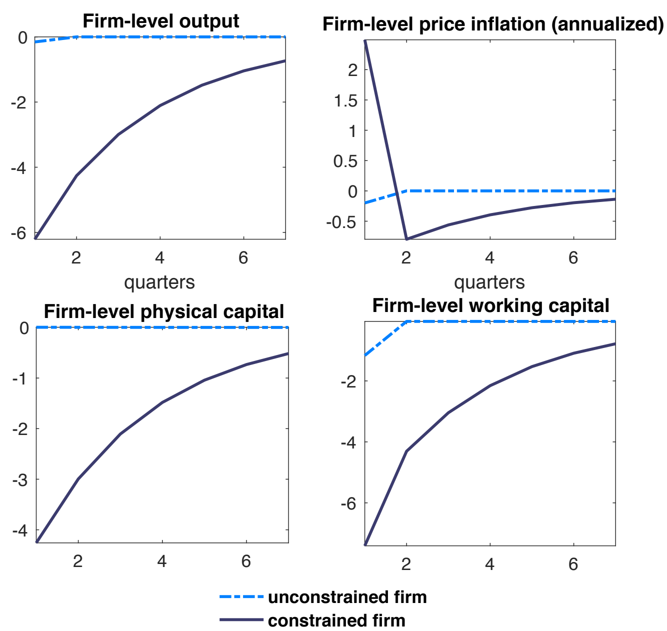



(b) Firm-level responses $\phi = 0.2$

Figure 9: Dynamic responses to a monetary tightening
Note: Y-axis: % deviation from steady-state. X-axis: quarters




(a) Aggregate responses




(b) Firm-level responses $\phi = 0.2$

Figure 10: Dynamic responses to a monetary tightening ($\alpha = 0.03$, Iacoviello (2005))
Note: Y-axis: % deviation from steady-state. X-axis: quarters

The third interesting finding in terms of aggregate monetary policy transmission is that a "price puzzle" may emerge when some of the firms in the economy face financial-frictions. Specifically, the aggregate price level may increase in response to a rise in the interest rate. In the basic version where all firms are financially-unconstrained price inflation always decreases. In the presence of credit frictions at the firm level however, if a monetary tightening reduces aggregate supply strongly enough (dotted red line in figure 12 (b)) together with aggregate demand (dotted black line in figure 12 (b)), the fall in aggregate activity may be associated in equilibrium with a rise in the price level. Such a case necessarily arises when the balance-sheet channel is dominant (figure 12 (a)). In this case, prices of constrained firms are pushed upwards by the monetary tightening. Thus, for high enough shares of constrained firms (e.g. larger than $\geq 20\%$ under baseline calibration), such positive pressures translate in an increase in the aggregate price level (figure 9, top panels). The bottom panels in figure 9 show how in this case the output of constrained firms is pushed strongly downwards, whereas their prices are pushed upwards.

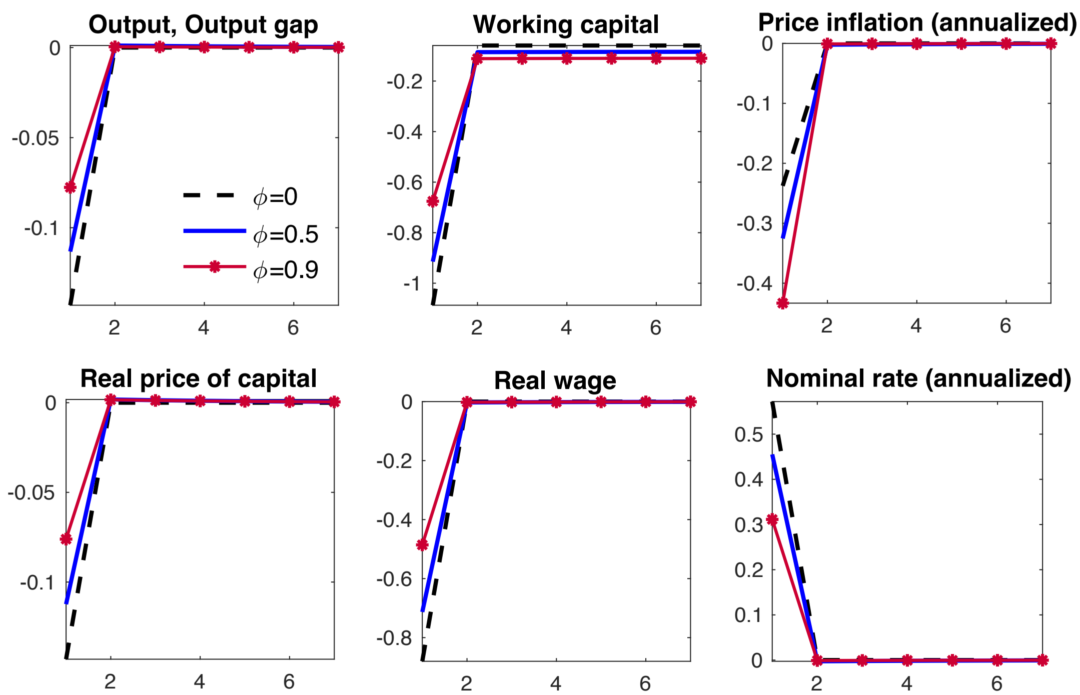
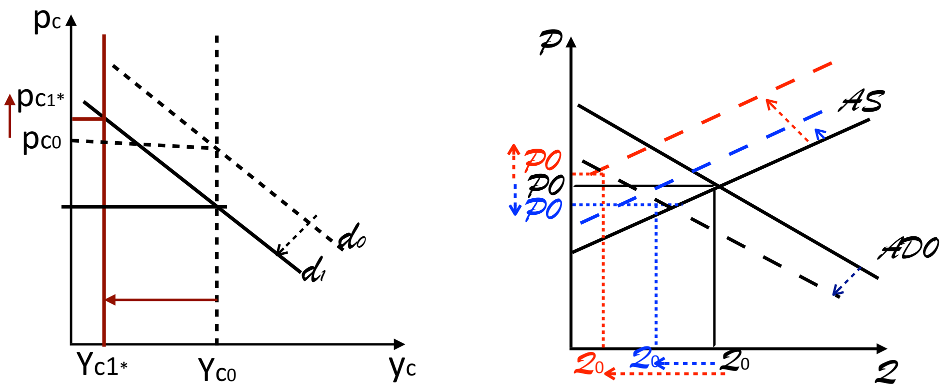



Figure 11: Dynamic aggregate response to a monetary tightening ($\alpha = 0.03$, $\theta^c = 45\%$)
Note: Y-axis: % deviation from steady-state. X-axis: quarters




(a) Firm-level

(b) Aggregate

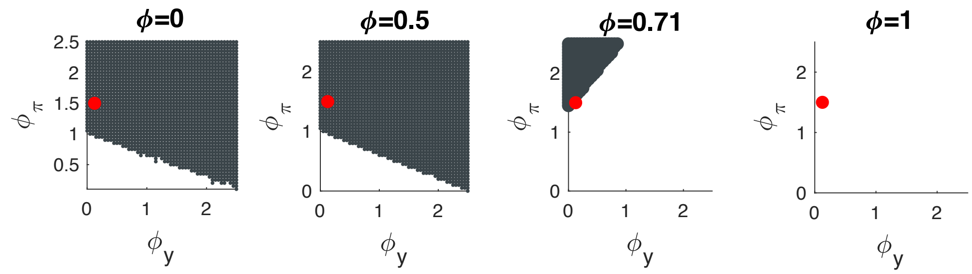
Figure 12: Strong balance sheet channel and the "price puzzle"

# 5 Monetary policy design

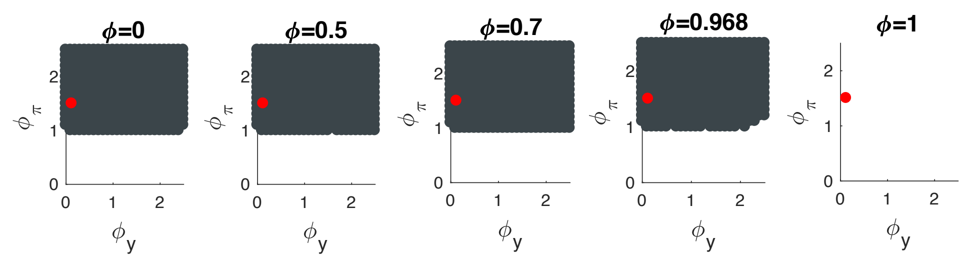
Since firms' credit frictions shape the effects of monetary policy, it is no surprise that they may ultimately affect as well the design of monetary policy. I first analyze how the financing frictions faced by firms may affect the stability properties of simple Taylor rules. I then focus on how monetary policy should take them optimally into account when deciding its response to business cycle fluctuations.

## 5.1 Equilibrium uniqueness and Taylor-rules

Given the changes in the transmission mechanisms of monetary policy, the properties that Taylor rules should satisfy to ensure equilibrium uniqueness may also be altered. I now study how these requirements change as the share of constrained firms increases. This question is important from a policy design perspective because, by ensuring that agents' expectations are anchored on an unique equilibrium path, the central bank avoids welfare losses due to sunspot fluctuations.




(a) Baseline calibration with sticky wages (dominant balance-sheet channel)




(b) Alternative calibration with $\alpha = 0.03$, $\theta^c = 0.45$ and flexible wages (dominant input-price channel)

Figure 13: Determinacy (in black) and indeterminacy (in white) regions
Note: Red dot indicates a standard Taylor rule with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$

According to the model, changes in the stability properties of simple Taylor rules reacting to price inflation and output are more likely to appear in an environment characterized by a strong balance-sheet channel. Specifically, under the baseline calibration with sticky wages characterized by a strong dominant balance-sheet channel such changes occur starting with a share of constrained firms in the economy around 70% (figure 13 (a)), whereas under the alternative calibration (i.e. with $\alpha = 0.03$ and $\theta^c = 0.45$) with flexible wages characterized by a dominant input-price channel they start occurring when this fraction is around 97% (figure 13 (b)).

### 5.2 Optimal monetary policy

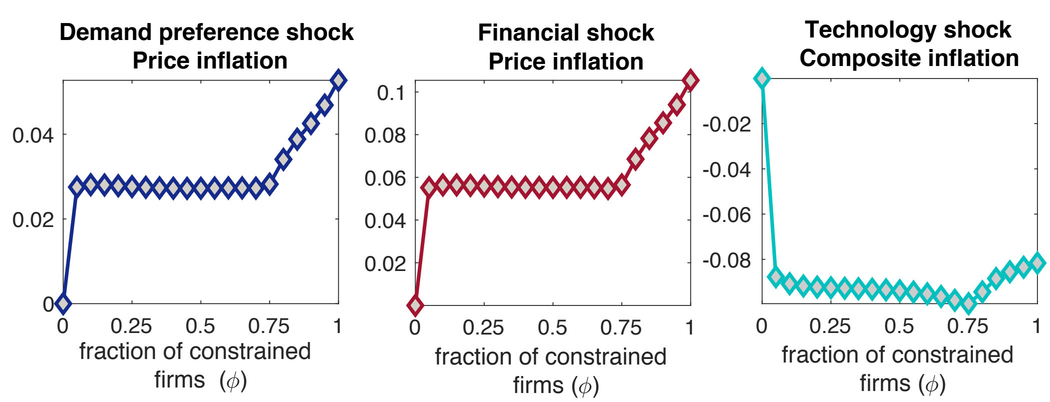
I now look how firms' financial constraints alter the optimal design of monetary policy. I assume employment is subsidized so as to correct for distortions associated to market power. Subsidies are financed by lump-sum taxes on households. The *flexible price* allocation in the credit-frictionless limit ($\phi = 0$) is thus the efficient benchmark at all dates (see page 60 in Appendix 10.2). I study optimal policy with commitment from a "timeless perspective" using the Linear-Quadratic approach. I focus on demand, technology and financial shocks. The latter is modeled as an exogenous variation in the collateral pledgeability ratio $\nu_t$.

**Welfare criterion** On page 70 in the Appendix 10.5, I derive a second order approximation to households' discounted utility fluctuations around steady state. Associated welfare losses, expressed as a share of steady-state consumption, equal:

$$\begin{aligned}\mathscr{L}_0 \approx & \frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\Big[\xi(1-\phi)(\pi_t^u)^2+\xi\phi(\pi_t^c)^2-(1-\sigma)\widehat{y}_t^2+\gamma^l\widehat{l}_t^2+\gamma^w(\widehat{\pi}_t^w)^2+ \\ & \gamma^{lcu}\Big(\phi\frac{L^c}{L}(\widehat{l}_t^c)^2+(1-\phi)\frac{L^u}{L}(\widehat{l}_t^u)^2\Big)-\gamma^{yc}\widehat{y}_t^c-\gamma^{kc}\widehat{k}_t^c+ \\ & +\gamma^{kcu}\Big(\frac{K^c}{K^u}\phi(\widehat{k}_t^c)^2+(1-\phi)(\widehat{k}_t^u)^2\Big)\Big]+t.i.p. \end{aligned} \tag{24}$$

where $\gamma^l, \gamma^w, \gamma^{lcu}, \gamma^{yc}, \gamma^{kc}, \gamma^{kcu} \geq 0$ defined in the Appendix 10.5 (page 70), $t.i.p.$ terms independent of policy, and $\gamma^{yc}$ and $\gamma^{kc}$ are small due to small steady-state distortions (table 7 on page 72 in the Appendix 2.11.5).

I use the welfare criterion consistent with the model (24) together with the first-order approximation of the equations describing the functioning of the decentralized economy to derive the equilibrium dynamics under optimal monetary policy with commitment. Details are deferred to the Appendix 10.5 on page 70. I study the responses of the economy to demand, technology and financial shocks under optimal monetary policy, namely when the policy rate is chosen so as to maximize household's welfare. For brevity, given empirical results presented in the next section, I focus exclusively on cases with a dominant balance-sheet channel.




Note: Y-axis: Effect on impact of a positive transitory shock, % deviation from steady-state. X-axis: share of constrained firms ($\phi$)

Figure 14: Departures from optimal policy regime in the credit-frictionless limit

**Importance of spillovers** Overall, transmission spillovers between constrained and unconstrained firms play an important role in the design of monetary policy. Specifically, optimal policy in the heterogenous case is not the simple weighted average of the one in the two polar cases (with only unconstrained and only constrained firms). Figure 14 (baseline calibration), shows that departures from the optimal monetary policy regime in the absence of credit frictions are relatively small unless the share of constrained firms is very large. A similar nonlinear pattern is obtained for the alternative calibration with $\alpha = 0.03$.

**Optimal policy response to demand preference shocks** Demand shocks affect only the efficient real interest rate, but not the efficient allocation. In the credit-frictionless limit (figure 15 for $\phi = 0$), monetary policy can replicate the efficient allocation by promising an aggressive response of the policy rate to variations in price inflation ("strict price inflation targeting"). Under this policy, whenever a shock $z_t$ pushes aggregate demand (16) in one direction, monetary policy $i_t$ offsets its effects by pushing it in the opposite one. Thus, in the absence of credit frictions, the central bank can perfectly insulate the economy from welfare losses caused by demand shocks.

Things are different when some of the firms in the economy are financially-constrained. First, their credit frictions distort the long-run allocation. Hence, the central bank has an incentive to engineer a positive variation in output irrespectively of the sign of the shock (i.e. engineer a positive variation in the output gap) so as to compensate for its long-run value being inefficiently low. Second, even if the central bank would like to insulate the allocation from the effects of such shocks, would not be able anymore. This is because, in contrast to the case without credit frictions, variations in the policy rate affect now directly both aggregate demand and aggregate supply. As a result, a variation in the policy rate aimed at offsetting the effects of the shock on aggregate demand has an additional supply-side effect. The latter induces a gap between the variation of the equilibrium allocation and the efficient one. As a result, in this case, demand shocks generally induce welfare losses under optimal policy, and the latter is conducted so as to minimize such losses. The nature of the optimal monetary policy regime, in particular its departure from price stability (the optimal regime in the absence of credit frictions), depends on structural parameters (e.g. figures 15 and 16).

When the balance-sheet channel dominates, a cut in the policy rate in response to a negative shock pushes up not only aggregate demand, but also aggregate supply via its positive effect on asset prices, and hence on collateral values. Thus, the cut in the policy rate that would offset the negative effects of the shock on aggregate demand, would also induce (inter-alia) a positive output gap and a decline in price inflation (because of additional supply-side effects).

Under the baseline calibration (figures 15) and the alternative one with $\alpha = 0.03$ (not shown), both characterized by strong balance-sheet channels, the positive supply-side effects of an interest rate cut are so strong, that the optimal response in the policy rate is very mild (and even slightly positive) so as to avoid strong deflationary pressures[29]. When the balance sheet channel is weaker (but still dominant), a policy rate cut has weaker negative supply-side effects on inflation. Hence, the policy rate declines more under optimal policy than in cases with a stronger balance sheet channel (figure 16)[30]. Importantly, because of the additional positive supply-side effects, the optimal decline in the policy rate is lower than in the absence of credit frictions. As a result, the optimal policy response is less likely to be constrained by the ZLB when there are constrained firms in the economy. Departures from strict price inflation targeting (the optimal policy regime in the absence of credit frictions) depend on structural parameters. In particular, a strong balance sheet channel (figure 15) implies small such departures, whereas a weaker one implies larger such departures (figure 16).

[29]Mirroring results are obtained for positive demand shocks (not shown).

[30]This is the sticky wage version of the second alternative calibration with $\alpha = 0.03$ and $\theta^c = 0.45$. With flexible wages, the real cost channel dominates under this calibration.

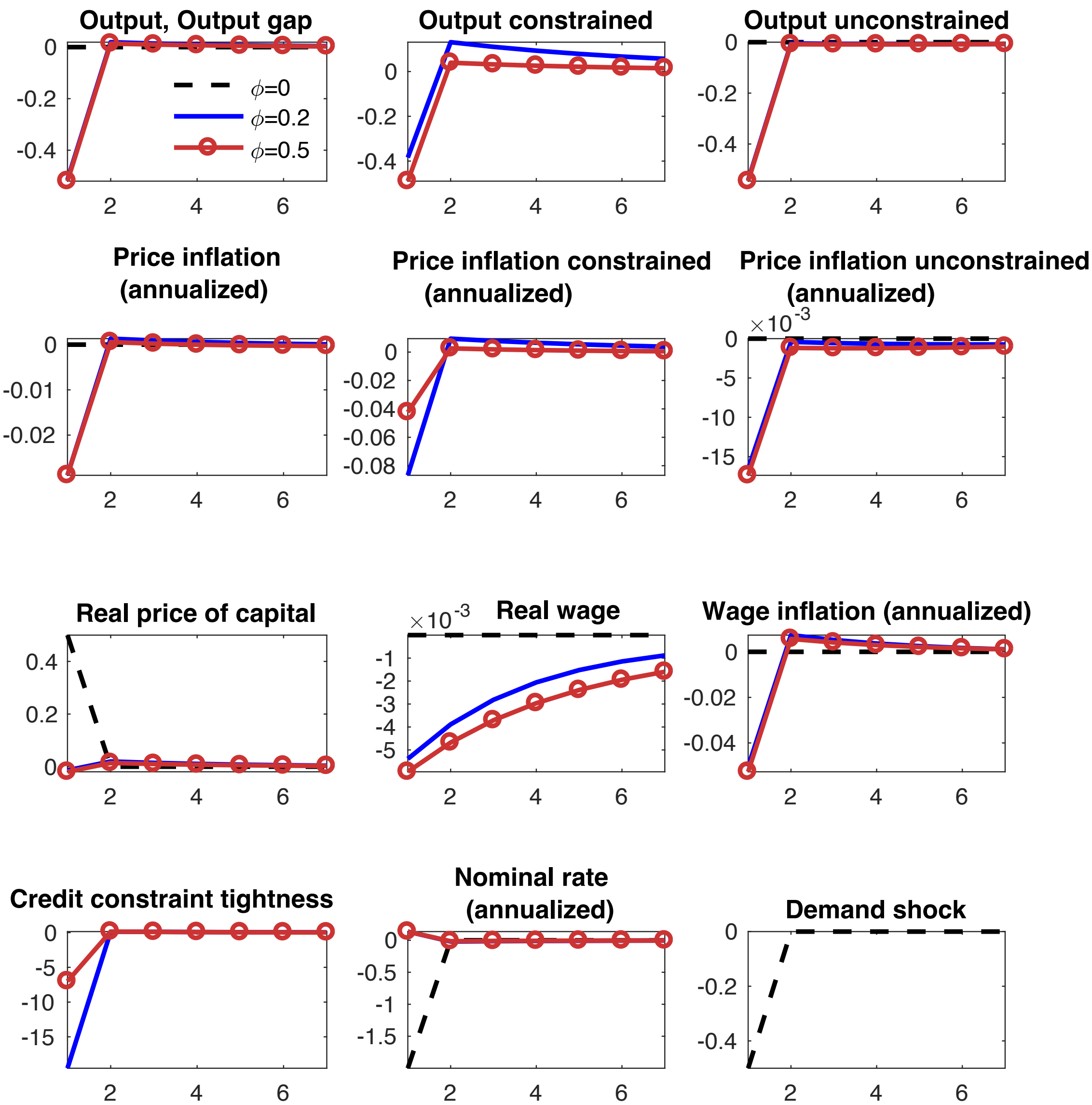



Figure 15: Optimal monetary policy: Dynamic responses to a transitory negative demand shock for different shares of constrained firms

Note: Y-axis: % deviation from steady-state. X-axis: quarters

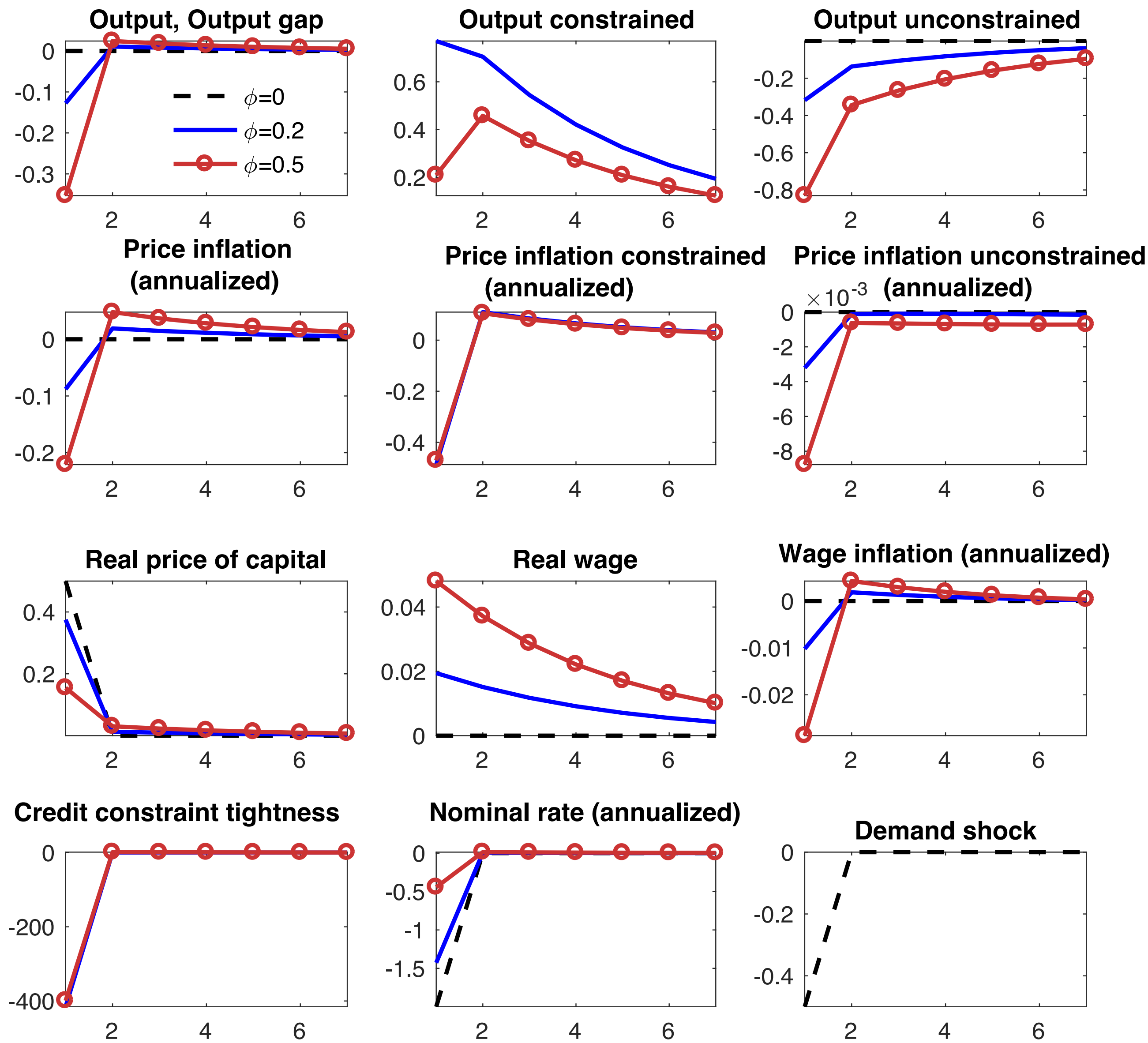



Figure 16: Optimal monetary policy: Dynamic responses to a transitory negative demand shock for different shares of constrained firms ($\alpha = 0.03$, $\theta^c = 0.45$)
Note: Y-axis: % deviation from steady-state. X-axis: quarters

**Optimal policy response to technology shocks** Technology shocks induce a welfare-tradeoff for monetary policy even in the absence of credit frictions. Specifically, monetary policy can no longer simultaneously stabilize price inflation, wage inflation and close the gap between output and its efficient level (Gal015), Chapter 5). This is because stabilizing both wages and prices, and hence the real wage, is incompatible with the (efficient) variation of the latter needed to make output vary one-for-one with its efficient level.

In the absence of credit frictions, the optimal monetary policy response entails closing perfectly the output gap at the expense of variations in price and wage inflation (figure 17 for $\phi = 0$). The simple rule approximating well optimal policy requires responding to both price and wage inflation ("composite inflation targeting") with the strength of each response a function of the degree of nominal rigidities in goods and labor markets. In response to a positive technology shock, the interest rate declines so as to prop up demand, and hence so as to make up for the difficulties of firms to quickly reduce prices. In equilibrium, this increase in demand allows firms to produce at efficient levels (despite price stickiness).

Both the trade-offs and the nature of monetary policy transmission change in the presence firms facing credit frictions. In particular, in response to a positive technology shock, the monetary authority may now have to either cut the policy rate as in the credit-frictionless case, or to mildly increase it. As in the case of demand shocks, the result depends on the strength of deflationary pressures of positive supply-side effects associated to the decline in the policy rate. When such pressures are strong, as in the case of the baseline calibration or the alternative one with $\alpha = 0.03$, the central bank may have to mildly increase the policy rate– in this case, if the central bank were instead to decline it, collateral asset prices would go up allowing constrained firms to produce more, and hence amplifying deflationary pressures (figure 17). When the balance-sheet channel is weaker (i.e. under the calibration with $\alpha = 0.03$ and $\theta^c = 0.45$), deflationary supply-side effects are weaker, and the policy rate declines under optimal policy (figure 18). But, as for demand shocks, because of the additional supply-side effects, the policy rate cut is weaker under optimal policy than in the absence of credit frictions.

**Optimal policy response to financial shocks** Financial shocks become relevant for the design of monetary policy in the presence of firms' credit frictions. A negative shock to collateral pledgeability allows constrained firms to produce less, and given their downward sloping demand schedule, it pushes upwards their prices. Thus, such financial shocks act as cost-push shocks. This result has already been put forward in models with only constrained firms in the context of collateral constraints by Carlstrom et al. (2011), and of agency costs by De Fiore and Tristani (2013).

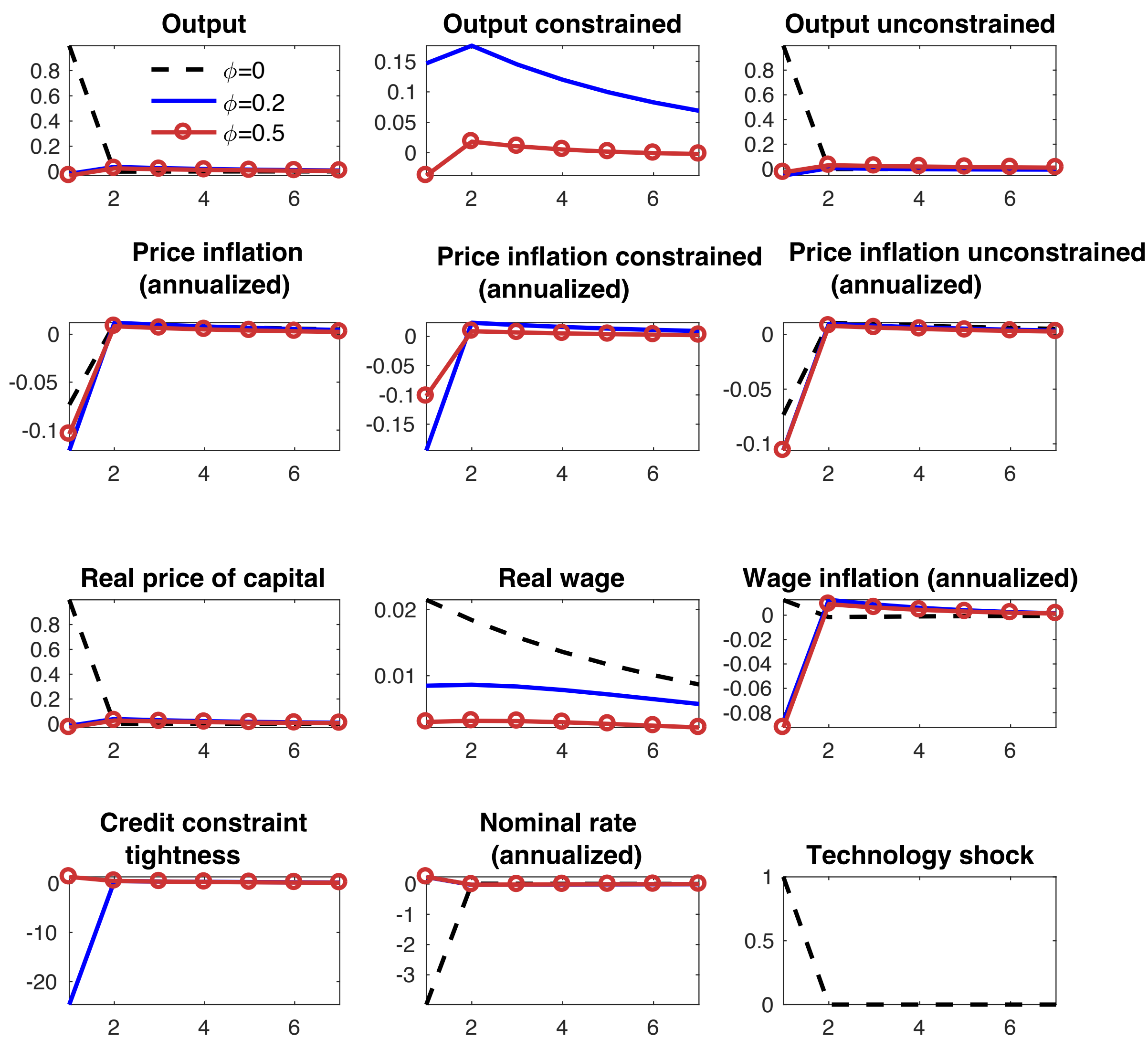



Figure 17: Optimal monetary policy: Dynamic responses to a transitory positive technology shock for different fractions of constrained firms
Note: Y-axis: % deviation from steady-state. X-axis: quarters

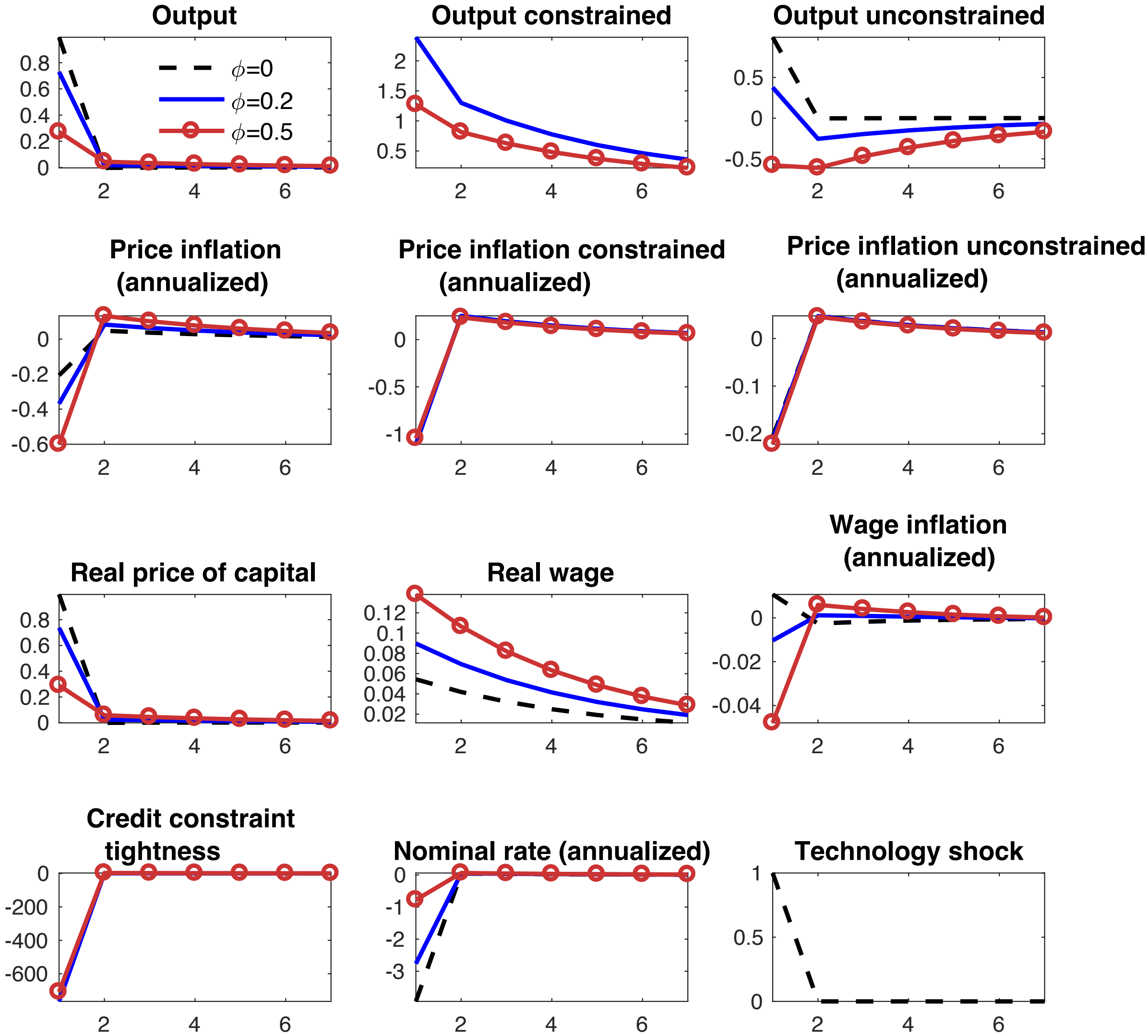



Figure 18: Optimal monetary policy: Dynamic responses to a transitory positive technology shock for different shares of constrained firms ($\alpha = 0.03$, $\theta^c = 0.45$)
Note: Y-axis: % deviation from steady-state. X-axis: quarters

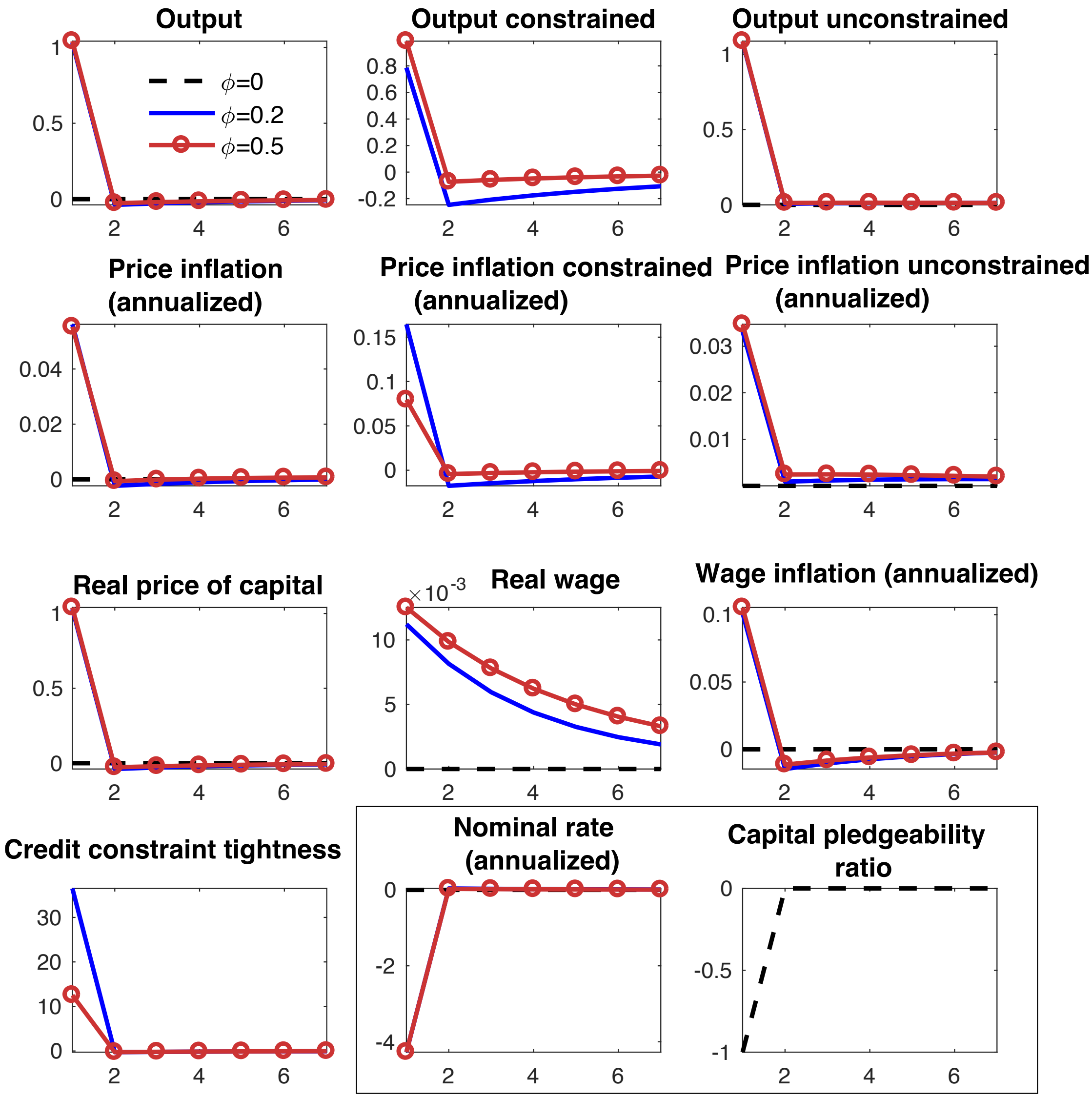



Figure 19: Optimal monetary policy: Dynamic responses to a transitory negative pledgeability ratio shock for different shares of constrained firms

Note: Y-axis: % deviation from steady-state. X-axis: quarters

As in Andres et al. (2013), but in contrast to Carlstrom et al. (2011), however, where collateral is not physical capital, the policy rate declines under optimal policy so as to prop up collateral asset prices, and hence to (partially) counteract the effect of the shock (figure 19). This is true under all three calibrations. Under the baseline calibration characterized by a strong dominant balance-sheet channel the decline is strong enough to actually increase collateral values in equilibrium. The latter effect allows constrained firms to produce above their steady-state levels and hence to contribute to the positive variation in the output gap. A similar result is obtained by De Fiore and Tristani (2013) in the context of costly-state verification. The latter however do not take into account the transmission spillovers between the two types of firms and the effect of steady-state distortions. Thus, relatively to these models, in the current heterogenous firm setup we can also see how the decline in the policy rate further induces unconstrained firms to expand production, and hence to push output above its inefficient steady-state level.

# 6 Empirical analysis

I now look whether the predictions of the model are in line with empirical evidence. In particular, I look whether the theoretical findings on firm-level transmission are corroborated by UK data. To do so, I extend the analysis in Cloyne et al. (2018)[31]. Specifically, relatively to this latter reference, I first study how monetary policy transmits to constrained firms depending on their characteristics (e.g. capital tangibility -as a proxy for pledgeability-, liquidity ratio). Cloyne et al. (2018) find that public UK firms with incorporation age less than 15 years and which do not distribute dividends reduce very strongly investment *on average* in response to a monetary tightening, whereas all others reduce it *on average* very weakly[32]. They also find that firm borrowing is highly correlated with collateral values, whereas the one of the latter group is not. They argue that these firms are most likely credit constrained, given that age is a good proxy for their track record in credit markets (hence, for credit access), and that not distributing dividends is a sign of a positive external finance premium (Fazzari et al. (1988), Mensa and Ljungqvist (2016), Jeenas (2018)). In the empirical analysis hereafter, I map the constrained set of firms in the model to this group, whereas the unconstrained one to all other firms in the sample.

[31] The empirical exercise is joint work with Ryan Banerjee (senior economist at the BIS).

[32] Their result is robust to controlling for size, asset growth, Tobin's Q, leverage or liquidity.

## 6.1 Methodology

The econometric methodology is based on an instrumental variable extension of the local projection method developed by Jorda et al. (2019). As in Cloyne et al. (2018) we use detailed financial statement data for publicly listed companies available from Thomson Reuters' WorldScope for the United Kingdom (table 2) and the five-year gilt yields for the interest rate[33]. The latter are instrumented with the series of monetary policy shocks constructed by Gerko and Rey (2017)[34]. These monetary shocks are obtained using the proxy-VAR/external instrument approach of Mertens and Ravn (2013) which uses movements in financial markets data (Short-Sterling Future contracts) in a short window around Bank of England policy rate announcements to isolate interest rate surprises. The sample spans from 1986 until 2018. Firms report for each fiscal year, but they do so in different months through the year. Thus, the data has a monthly dimension and refers to activity throughout the reporting year. All balance-sheet variables are converted to real values using the aggregate GVA deflator for the United Kingdom. For each observation, the interest rate is recorded at the end of the reporting month, and the series of monetary shocks used as instruments are also summed up to refer to the particular fiscal year. The asset tangibility ratio (used as a proxy for the capital pledgeability ratio in the analysis) is the ratio of tangible capital in total capital, where the latter is the sum of tangible capital and of an estimate of intangible capital computed using the methodology in Peters and Taylor (2017).

Baseline specification used to estimate the dynamic effects of a monetary tightening is a set of panel local projections of the form

$$X_{i,t+h} - X_{i,t-1} = \gamma_i^h + \sum_{g=1}^{G} \beta_g^h \cdot I[Z_{i,t-1} \in g] R_t + \sum_{g=1}^{G} \alpha_g^h \cdot I[Z_{i,t-1} \in g] + \varepsilon_{i,t+h} \tag{25}$$

with the dependent variable $X$ the variable of interest (investment or working capital), $h$ the number of (fiscal) years after the shock, $Z_{i,t-1}$ is a set of firm characteristics and the indicator function takes a value of 1 if firm characteristics fall in that particular firms's group. We include firm-fixed

[33] Variation in investment ratio winsored at 1%, variation in working capital at 5%, leverage at 1%.

[34] As in Cloyne et al. (2018), we drop firms within the finance, insurance, real estate and public administration sectors. Since the sample contains only public firms, to the extent that financial constraints are likely to be tighter for private firms than for public firms, the fraction of constrained firms in the UK economy, and hence the strength of spillover effects from constrained to unconstrained firms, are likely underestimated in our analysis. The use of monetary policy shocks ensures that the estimated effect of the variation in interest rate is not driven by other macro factors (namely, by endogenous components of monetary policy rule). We use the series of monetary policy shocks used in Cloyne et al. (2018) which was kindly sent to us by Paolo Surico.

Table 2: Firm-level balance-sheet data (WorldScope)

| Variable ($X_{i,t}$) | Definition | WorldScope series |
|---|---|---|
| Investment | Investment rate = capital expenditures/ lag of property, plant and equipment | 04601/ 02501 |
| Working capital | log (working capital) | 03151 |
| Age | - | 18273 |
| Dividends paid | first lag | 04551 |
| Tangible capital | property, plant, equipment | 02501 |
| Leverage | total-debt/total-assets, first lag | 03255/02999 |
| Liquidity ratio | cash & short-term investments/total assets, first lag | 02001/02999 |
| Short term debt | first lag | 03051 |
| Total debt | first lag | 03255 |

effects $\gamma_i^h$ and monthly dummies[35]. The interest rate interacted with the dummies is instrumented by the monetary policy shock (also interacted with the dummies). The coefficients of interest are the $\beta_g^h$ which measure how the effect of a monetary tightening on firm investment $h$ (fiscal) years after the shock depends on firm's characteristics. As in the model, results are reported for a 25bps shock. Errors are clustered at the firm-level.

## 6.2 Results

We start by running the set of regressions for the two groups "constrained" and "unconstrained" for investment (as in the original paper) and working capital. The $\beta_h^g$ coefficient for the two groups at different horizons are plotted in figure 20. Firms in the constrained group are found to reduce their working capital and investment more than unconstrained ones in response to a monetary tightening. Through the lenses of the model, this implies that in the UK monetary policy affects the activity of constrained firms relatively stronger via the balance-sheet and debt-deflation channels.

[35]Firm fixed effects capture permanent differences in investment behaviour across firms.

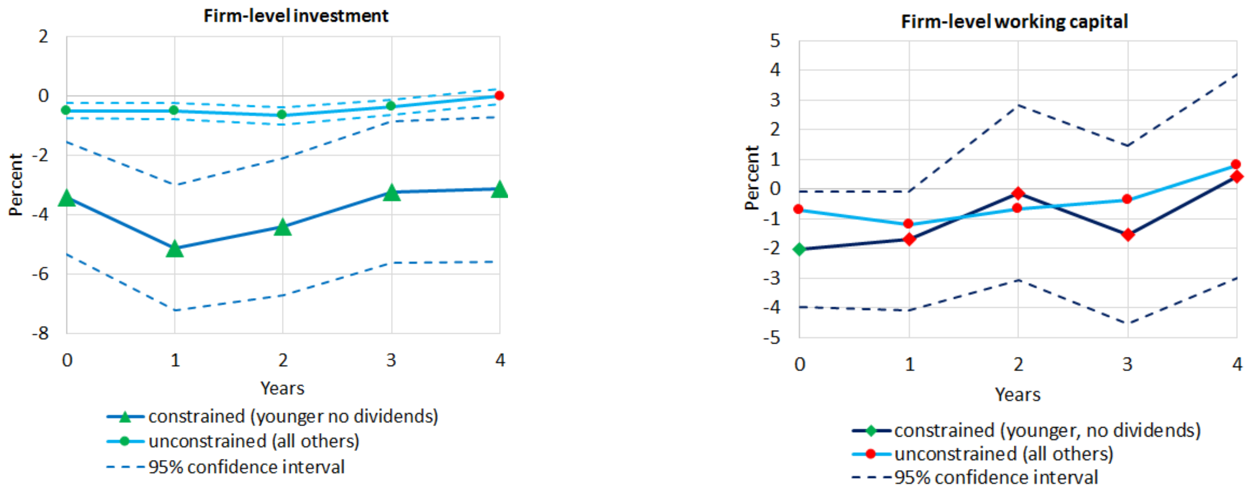
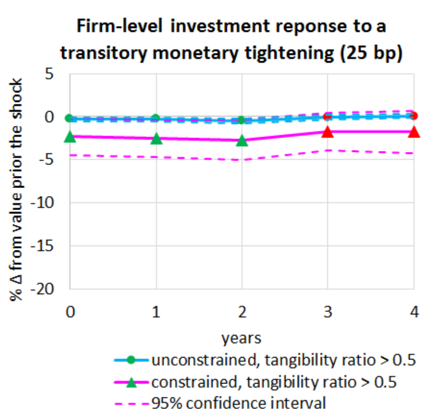



Figure 20: Estimated responses to a monetary tightening in the UK

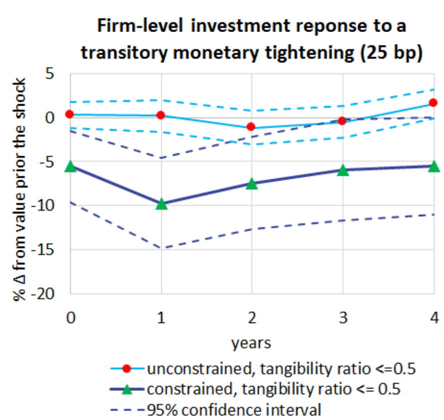
Now we try to understand transmission within the constrained set in more detail, by looking at the effect of parameters analyzed in the theoretical section: capital tangibility and liquidity ratio ("net worth"). We also try to identify instances where the activity of constrained firms is steered in a positive unconventional direction by a monetary tightening (i.e. a stronger input-price channel). We first explore how the responses of constrained firms to monetary policy depend on the pledgeability of their physical capital as collateral. As more tangible capital is expected to be more pledgeable as collateral, we use the former as a proxy in the empirical analysis. We start by splitting the constrained set at each date in two groups based upon their capital tangibility ratio in the previous fiscal year. The set of regressions (25) is thus specified based on the following three groups $g$: constrained firms with high capital tangibility, constrained firm with low capital tangibility and unconstrained firms.

Figure 21 plots the $\beta_g^h$ coefficients in the case of investment for the two subgroups in the constrained set and for the unconstrained group. The left panel compares the response of constrained firms with high capital tangibility (purple line) with the one of unconstrained ones (light blue), whereas the right panel compares the response of constrained firms with low capital tangibility (navy line) with the one of unconstrained firms. The figure shows that firms in the constrained group with low capital tangibility respond the most. If not paying dividends is a good proxy for being financially constrained (and hence, assuming that all firms in this group are constrained is a good approximation), this result implies that the strength of the balance sheet channel decreases with the pledgeability of capital.

Table 3 shows that these results hold more generally. Specifically, it shows the results of a regression with "constrained" and "unconstrained" firms as the two groups $g$, and an interaction term between being in the constrained group and the capital tangibility ratio in the fiscal year prior to the shock. The sign of the latter coefficient of the interaction term is positive and significant, implying that a higher capital tangibility ratio is associated to a weaker decline in investment, and hence to a relatively weaker balance-sheet channel. This result is in line with the theoretical predictions discussed in the previous section (figure 27 on page 65 the Appendix). In the model this result arises because a decrease in the pledgebility of capital implies a decrease in the share of working capital credit in total credit, and hence of a decrease in the relative strength of the real cost channel. Importantly, a model with only investment credit as the one in Iacoviello (2005) would imply the opposite. Thus, our empirical results may also be taken as evidence of the importance of working capital credit for constrained firms in the sample.




(a) High tangibility




(b) Low tangibility

Figure 21: Estimated investment responses to a monetary tightening (25bp)

Also in line with theoretical predictions is the positive highly significant correlation between capital tangibility and leverage in the constrained group shown in table 4. This correlation implies that, as in the model, constrained firms with low capital pledgeability tend to be less levered[36]. Furthermore, consistent with theoretical predictions, there is no such significant correlation in the group of old firms which distribute dividends (the most likely "unconstrained" group).

[36]In the model, lower capital pledgeability is associated to lower steady-state leverage ratios of constrained firms.

Table 3: Effect of the asset tangibility ratio on the magnitude of the response of credit constrained firms to monetary policy in the UK

| | Investment ratio | | | | |
|---|---|---|---|---|---|
| | (h=0) | (h=1) | (h=2) | (h=3) | (h=4) |
| r5 Indy | -5.925 *** | -11.003*** | -9.535*** | -7.068*** | -6,00*** |
| r5 Indytarlag | 4.067** | 9.13*** | 8.401*** | 6.038*** | 4.4326 ** |
| r5 Iu | -0.506*** | -0.489*** | -0.659*** | -0.375** | -0.025 |

Regression: $I_{i,t+h} - I_{i,t-1} = \gamma_i^h + \beta_{indy}^h \cdot Indy \cdot R_t + \beta_{indyta}^h \cdot Indytarlag \cdot R_t + \beta_{iu}^h \cdot Iu \cdot R_t + \alpha_{indy}^h \cdot Indy + \alpha_{indyta}^h \cdot Indytarlag + \alpha_{iu}^h \cdot Iu + \alpha_{indy}^h \cdot Indy + i.m + \varepsilon_{i,t+h}$
where $Indy$ is an indicator variable which equals 1 if the firm is younger than 15 years and does not distribute dividends, $Indytarlag$ is an interaction variable with the (lagged) tangibility ratio of firm's assets and $Iu$ is an indicator variable which takes the value 1 when $Indy = 0$.
The interest rate is instrumented with the series of monetary policy shocks.

* $p < .1$, ** $p < .05$, *** $p < .01$

Table 4: Correlation asset tangibility ratio and leverage (significance level)

| | |
|---|---|
| constrained (younger-no dividends) | **0.1037** (0) |
| older-dividends (i) | 0.001(0.8976) |

* $p < .1$, ** $p < .05$, *** $p < .01$

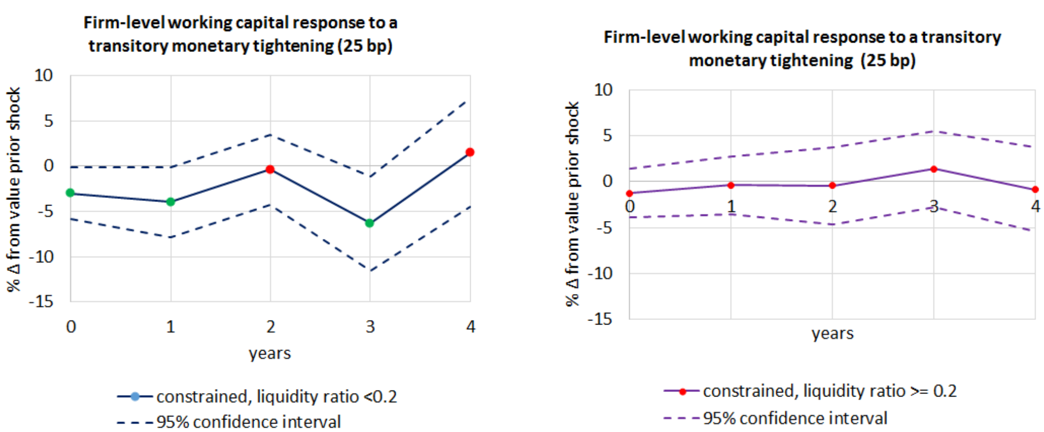
We now look how the response of constrained firms to monetary policy depends on their ability to finance production with their own net worth, otherwise stated how the tightness of their credit constraint affects the response to monetary policy[37]. We use the liquidity available prior to the shock as a proxy for the funds that the firm could use to finance production aside credit. As in Cloyne et al. (2018) we normalize liquidity by the size of total assets. We split the constrained set at each date in two groups based upon their liquidity ratio in the previous fiscal year. The set of regressions (25) is thus specified based on three subgroups $g$: constrained firms with high liquidity ratio, constrained firm with low liquidity ratio and "unconstrained" firms. We use 20% to define the threshold between low and high liquidity ratios[38]. Figure 22 plots the $\beta_g^h$ coefficients in the case of working capital for the two subgroups in the constrained set. As in the model, firms that can use less of their own funds to finance production (here, firms with a low liquidity ratio) respond more to monetary policy.

In the theoretical analysis we saw that a strong input price channel is favored by a high share of

[37]Constrained firms with higher net worth need less credit, and hence their financial constraint is expected to be looser.

[38]Around 2/3 of observations are below this threshold. Results are robust and differences between the two subgroups more striking when defining a lower threshold of 10% which splits the constrained group more evenly in two sub-groups.

working capital credit in total credit. We do not have information on working capital credit, but we do on short-term debt and we use the latter as a proxy. We look at the limiting case of firms with only short-term debt and focus on the ones with a low pledgeability of their capital. Specifically, we define two groups $g$ one including the latter group of firms, and the other all firms in the economy and run the regression in (25). The $\beta^g$ coefficients are plotted in the left panel of figure 23. The ones for the constrained groups with short-term debt only and capital pledgeability ratio lower than 0.9 are plotted in navy blue, whereas the ones for all other firms in light blue[39]. The figure shows that investment by firms in the first group is steered by a monetary tightening in a positive unconventional direction on impact in line with a relatively stronger input-price channel. The right panel of figure 23 shows that the model makes a similar prediction in this special case. Specifically, both the theoretical responses of investment and output are steered in an unconventional positive direction when the firm has only working capital credit and a low capital pledgeability ratio[40].




(a) Low liquidity-ratio (b) High liquidity-ratio

Figure 22: Estimated working capital responses to a monetary tightening (25bp)

More generally, by running the regression in (25) with two groups of firms constrained and unconstrained and an interaction term between being constrained and the lagged value of short term to long term ratio, we obtain a positive significant coefficient for the latter (table 5). This implies that, to the extent that short term debt is a good proxy for working capital credit, as in the model, a higher share of short term debt is associated to a weaker response of a constrained firm to

[39]Initially, I run a regression with four groups:. However, results for the constrained group with only short term debt and low capital pledgeability are the same as for the simpler regression.

[40]The pledgeability ratio is set low enough for a set of firms to be constrained in equilibrium under the alternative calibration following Iacoviello (2005).

a monetary tightening[41].

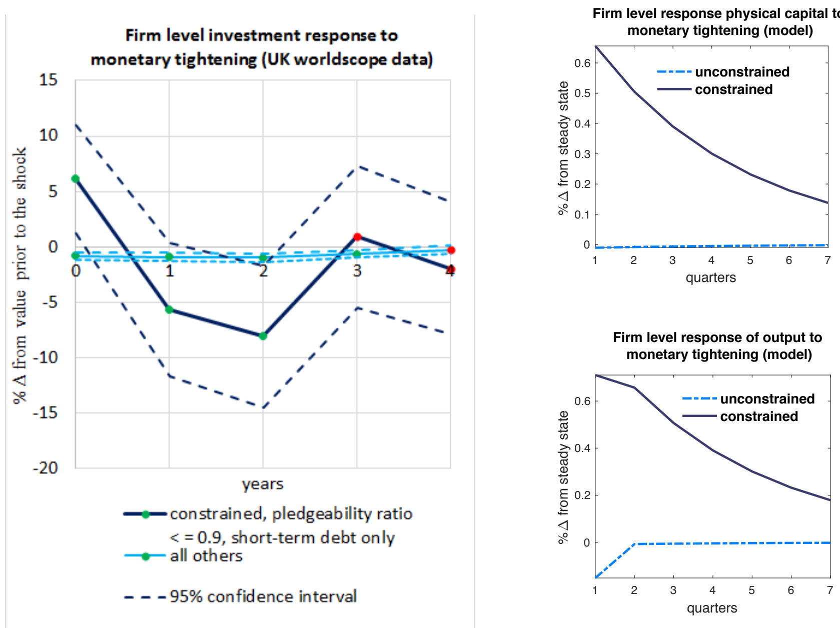



Figure 23: Estimated investment responses to a monetary tightening (left). Theoretical investment responses to a monetary tightening; alternative calibration 2 with flexible wages, $\theta^c = 1$ i.e. 100% of physical capital financed by equity and $\nu = 0.2$ i.e. 20% of physical capital is pledgeable as collateral (right)

Table 5: Share of short-term debt in total debt $sh/lt - lag$ and the response of credit constrained firms to monetary policy in the UK

| | Investment ratio (h=0) |
|---|---|
| r5 Indy | -5.037*** |
| r5 $Indy - sh/lt - lag$ | 4.81** |
| r5 Iu | -0.477*** |

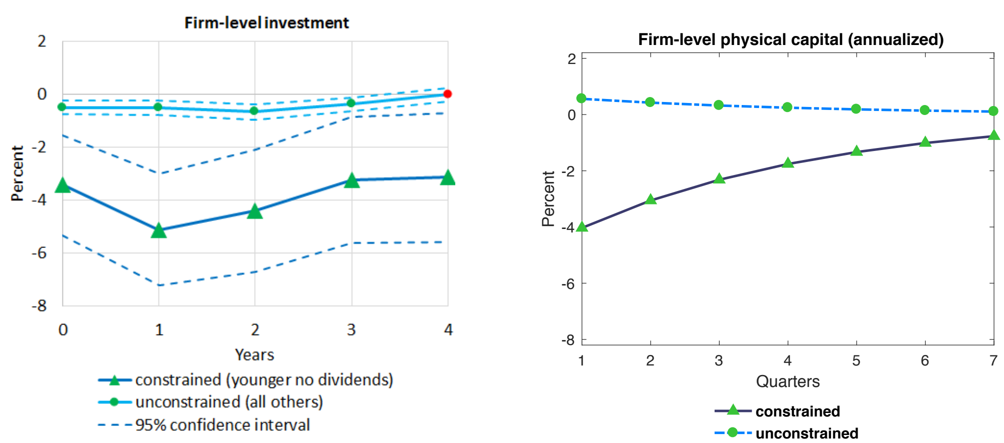
* $p < .1$, ** $p < .05$, *** $p < .01$

[41]Regression: $I_{i,t+h} - I_{i,t-1} = \gamma_i^h + \beta_{indy}^h \cdot Indy \cdot R_t + \beta_{indyta}^h \cdot Indytarlag \cdot R_t + \beta_{iu}^h \cdot Iu \cdot R_t + \alpha_{indy}^h \cdot Indy + \alpha_{indyta}^h \cdot Indytarlag + \alpha_{iu}^h \cdot Iu + \alpha_{indy}^h \cdot Indy + i.m + \varepsilon_{i,t+h}$ where $Indy$ is an indicator variable which equals 1 if the firm is younger than 15 years and does not distribute dividends, $Indytarlag$ is an interaction variable with the (lagged) tangibility ratio of firm's assets and $Iu$ is an indicator variable which takes the value 1 when $Indy = 0$. The interest rate is instrumented with the series of monetary policy shocks as described in table 2.

# 7 Use of the model for policy analysis

The model can be used to give a structural interpretation to the estimated dynamic responses to a monetary tightening shown in the previous section. According to those estimations (reported again in the left panel of figure 24), financially-constrained firms reduce their activity stronger than unconstrained ones. By setting the capital leverage ratio of constrained firms at $\theta^c = 0.32$ under the alternative calibration with $\alpha = 0.03$, we can match reasonably well the empirical firm-level responses (right panel of figure 24).

The empirical results can be explained through the lenses of the model by a dominant balance-sheet channel. Specifically, as the central bank rises the policy rate, collateral asset values decline, and hence constrained firms are compelled to cut strongly production. The strong adverse effects on constrained firms further spill over to unconstrained ones, benefiting the latter. Specifically, the strong decline in production by constrained firms decreases competition on output markets for the unconstrained ones, and implies strong negative pressures on input prices (because of the strong decline in input demand). These positive spillover effects via output and input markets partially counteract the negative effects of the monetary tightening on the firms which do not face financing constraints. Hence, they respond only mildly in equilibrium.




(a) Estimated responses (UK) (b) Theoretical responses

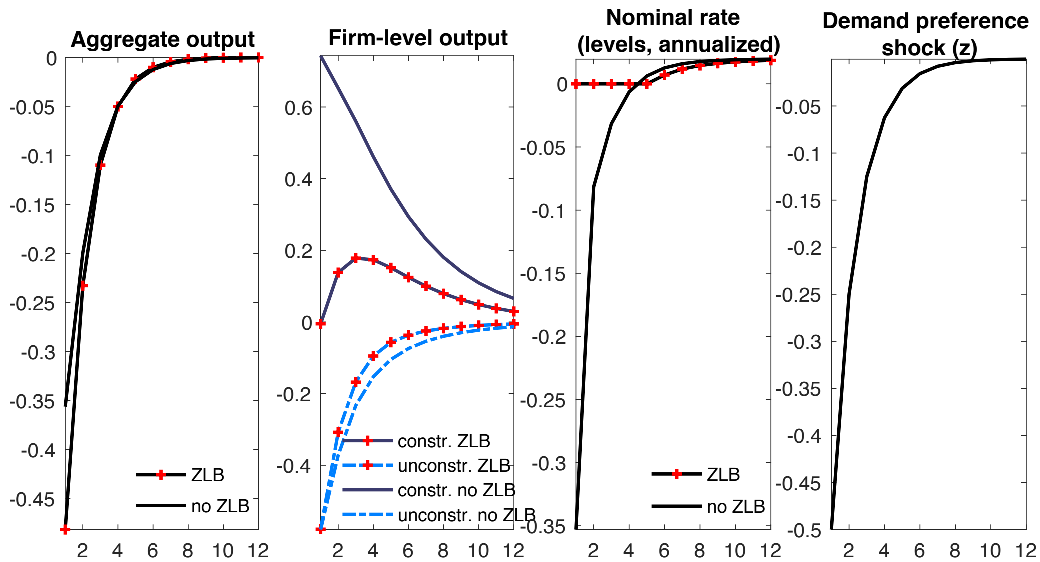
Figure 24: Response of investment to a monetary tightening (25 bps)

The model may also help us understand better how monetary policy transmits in the current low interest rate environment where the probability that the policy rate hits the ZLB is high. In particular, the model outlines that in economies with strong dominant balance-sheet channels (such as the UK, the US, and most likely the Euro Area), the presence of an effective ZLB on the policy rate particularly hurts financially-constrained firms, while unconstrained ones may benefit. To see why, note that in the absence of the ZLB, constrained firms are more negatively affected than unconstrained ones in response to a monetary tightening, but more positively affected in response to a monetary loosening. Thus, in the absence of the ZLB, the relatively gains and losses for these firms compensate over the business cycle.

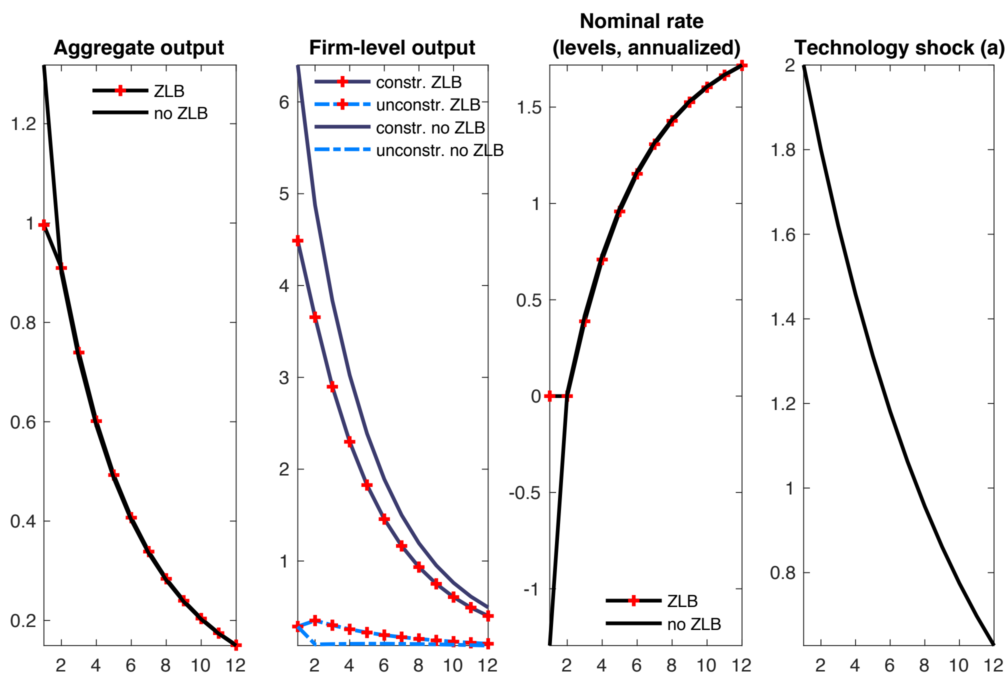
Since the ZLB limits the more positive effects of monetary policy when an interest rate cut is warranted, but the more adverse effects remain unchanged, on average over the business cycle, constrained firms end up particularly hurt. Furthermore, because of spillover effects, unconstrained firms benefit. Specifically, since constrained firms are affected less positively by a cut in interest rates, the associated negative spillovers to the unconstrained ones are also lower. As a result, the net positive effects of monetary policy on unconstrained firms in times where a monetary loosening is warranted are stronger. Thus, despite the stronger decline in aggregate activity when the ZLB binds, unconstrained firms may not be affected by the latter in equilibrium, or they may even end up producing more.

Consistently, figure 25 illustrates the effects of the ZLB on constrained and unconstrained firms when the economy is hit by demand, technology and financial shocks. Result are conditional on the Taylor rule considered in our analysis which proxies the way monetary policy is conducted in practice. The time preference parameter is set to 0.995 so as to imply a (lower) long-run interest rate of 2% in annualized terms (compared to 4% under baseline calibration), and the sizes of the shocks are set such that the model economy hits the ZLB. Results are reported for the UK calibration. For all three types of shocks it can be observed how the ZLB limits the positive effects of the decline in the policy rate on the production of constrained firms. It also shows how, despite the ZLB, the shock affects production of unconstrained firms either the same or even less than in its absence.The latter result is explained by the smaller negative spillover effects of monetary policy transmission to constrained firms.

Finally, one could further use the model more generally to study how optimal policy departs from the standard prescriptions derived for the credit-frictionless limit. According to the model, when ignoring the ZLB on the policy rate, such departures are only marginal in the case of the UK. Specifically, they are of order $-0.08\%$ (in annualized terms) from strict price inflation targeting for a transitory standard negative demand shock (i.e. implying an annualized variation in the efficient rate of 1%), of order 0.16% for a transitory 1% negative shock to the capital pledgeability ratio, and of order −0.025% from composite inflation targeting for a transitory positive technology shock.

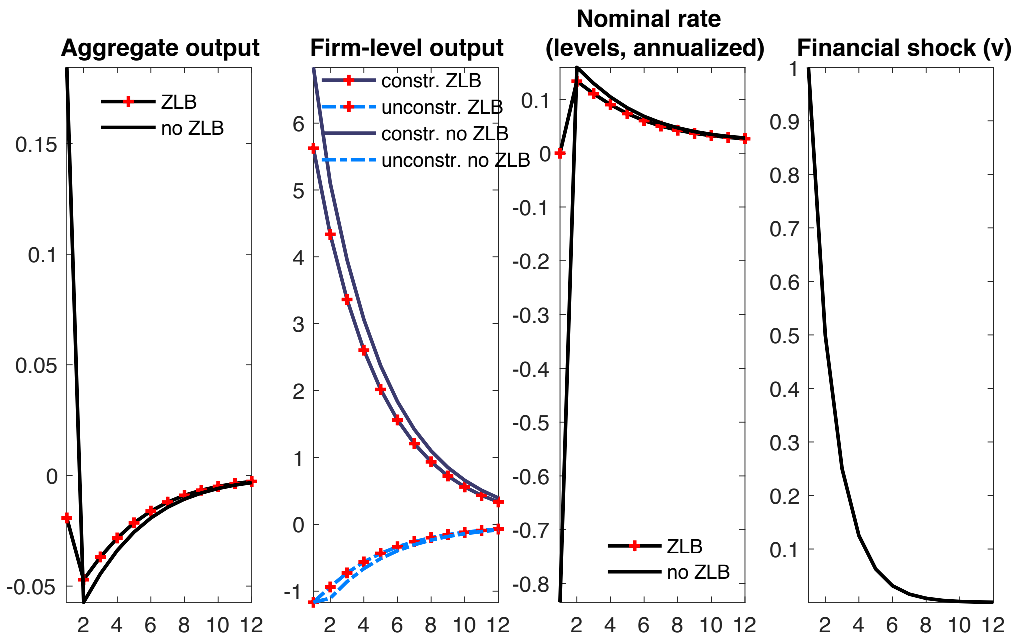



(a) Demand preference shock




(b) Technology shock

Figure 25: Responses to shocks subject to the ZLB




(c) Financial shock

Figure 25: Responses to shocks subject to the ZLB

Relatively to the credit-frictionless benchmark, the policy rate declines less in response to both demand and technology shocks under optimal policy (left and right panels in figure 26). Furthermore, as already pointed out in the theoretical section on optimal policy design, the policy rate declines in response to the financial shock in order to prop up collateral values (middle panel in figure 26).

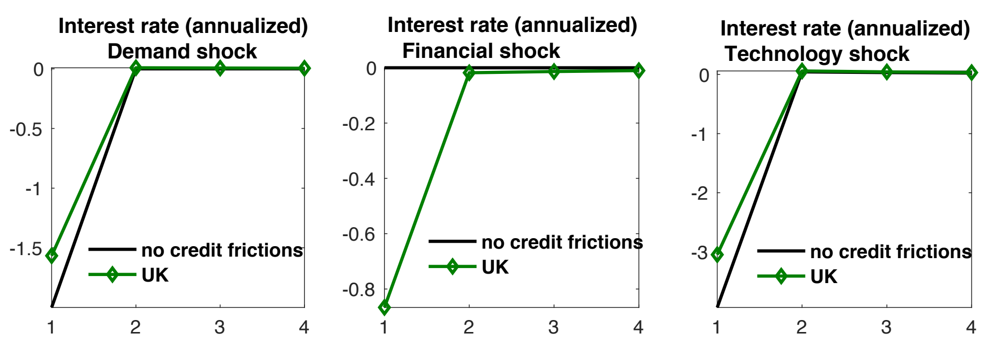



Figure 26: Optimal policy rate response - UK calibration and credit-frictionless setup

Going forward, since the low interest rate environment seems here to stay, it would be important to use the model to understand how monetary policy should be optimally conducted in this environment in conjunction with other available policy instruments, while taking explicitly into account the interactions between constrained and unconstrained firms in the economy.

## 8 Conclusions

In this paper I propose a stylized model to study how transmission and optimal design of monetary policy change with the share of firms facing financing constraints. The analytical framework is an extension of the Rotemberg version of the basic New Keynesian model with working capital paid in advance and physical capital in fixed aggregate supply. I find that credit frictions activate a number of additional transmission mechanisms in the New Keynesian setup through which monetary policy affects the supply-side of the economy in opposite directions. For instance, a monetary tightening depresses production by constrained firms by pushing downwards collateral values and expands it by reducing prices of inputs financed against collateral. These indirect effects spill over to unconstrained firms via input and output markets, rendering firms' heterogeneity in term of access to credit relevant for monetary policy. In equilibrium, credit frictions may both amplify or dampen the reaction of output to monetary policy depending on structural parameters, and generate "price puzzles".

Changes in transmission further translate into changes in the optimal design of monetary policy, but the latter may not be quantitatively significant unless the share of constrained firms is very high. Empirical evidence based on UK data corroborates the predictions of the model on how monetary policy affects constrained firms given the tangibility of their assets and their liquidity ratio, and uncovers a set of constrained firms whose output is steered in an unconventional direction by monetary policy. In the end, the analytical setup is used to point out that in the current low interest rate environment in countries with a dominant balance-sheet channel such as the UK (and also, the US and most likely the Euro Area), financially-constrained firms are particularly hurt, whereas unconstrained ones may benefit when the policy rate hits the ZLB.

Ongoing work focuses on bringing supporting evidence for the two new channels of monetary policy transmission identified in the theoretical analysis, namely the "input price channel" and the "spillover channel". Moreover, the empirical analysis is being extended for the US and the Euro Area.

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# 10 Appendix

## 10.1 Log-linear approximation

The analysis focuses on the first order approximation of equilibrium dynamics in the vicinity of the non-stochastic zero-inflation steady-state. Shocks are small enough for supply to be non-rationed and credit constraints to remain tight. Notation is standard: small caps stand for the log-levels, $(\widehat{\ })$ for the log-deviation from steady-state, while the absence of a time subscript denotes a steady-state value.

Households' behavior is described by the consumption/saving decision:

$$\widehat{c}_t = E_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\Big(\widehat{i}_t - E_t\{\pi_{t+1}\}\Big) + \frac{1}{\sigma}(1-\rho_z)z_t \tag{26}$$

and the (aggregate) labor supply:

$$\varphi\widehat{l}_t + \sigma\widehat{c}_t = \widehat{\omega}_t, \tag{27}$$

where $\widehat{\omega}_t$ is the log-deviation from steady-state of real wage. Households' behavior is not affected by the presence of credit frictions on the supply-side of the economy. Demand for goods in each set (constrained and unconstrained) equals:

$$\widehat{y}_t^s = -\varepsilon\widehat{pp}_t^s + \widehat{y}_t, \qquad s = \overline{u, c} \tag{28}$$

where $\widehat{pp}_t^s \equiv \widehat{p}_t^s - \widehat{p}_t$.

On the supply-side, firms' behaviour is described by their price-setting decision:

$$\pi_t^s \approx \beta E_t\{\pi_{t+1}^s\} + \lambda^s\Big(\widehat{mc}_t^s - \widehat{pp}_t^s + \frac{\lambda^{1,s}}{1+\lambda^{1,s}}\widehat{\lambda}_t^{1,s}\Big) \tag{29}$$

with $\lambda^s \equiv \frac{\varepsilon}{\xi\mathscr{M}}\frac{P^s}{P}$, $\widehat{\lambda}_t^{1,u} = 0$ and $\widehat{mc}_t^s \equiv \widehat{\omega}_t + \widehat{l}_l^s - \widehat{y}_t^s$, their production level:

$$\widehat{y}_t^s = a_t + \alpha\widehat{k}_t^s + (1-\alpha)\widehat{l}_t^s,$$

their capital demand:

$$\begin{aligned}\widehat{\varrho}_t^k &\approx \beta\Big(1+\lambda^{1,s}\nu\Big)E_t\{\widehat{\varrho}_{t+1}^k\} + E_t\{\widehat{\Lambda}_{t,t+1}\} - \lambda^{1,s}(1-\theta^s)\Big(\widehat{i}_t - E_t\{\pi_{t+1}\} + \widehat{\varrho}_t^k\Big)\\ &+ \beta\frac{MC(i)(1+\lambda^1(i))\alpha Y(i)/K(i)}{Q^k/P}\Big(E_t\{\widehat{y}_{t+1}^s\} - \widehat{k}_{t+1}^s + E_t\{\widehat{mc}_{t+1}^s\}\Big)+\\ &+ \beta\lambda^{1,s}vE_t\{\widehat{\nu}_{t+1}\} + \lambda^{1,s}\Big[\beta\frac{MC(i)\alpha Y(i)/K(i)}{Q^k/P} + \beta\nu - (1-\theta(i))\Big]E_t\{\widehat{\lambda}_{t+1}^{1,s}\}\end{aligned}$$

where $\widehat{\varrho}_t^k \equiv log(\frac{Q_t^k}{P_t}) - log(\frac{Q^k}{P})$, $E_t\{\widehat{\Lambda}_{t,t+1}\} \approx E_t\{\Delta z_{t+1}\} - \sigma E_t\{\Delta\widehat{c}_{t+1}\}$ and, for firms in the constrained group $i \in \Theta^c$, additionally by their credit collateral constraint:

$$\widehat{\omega}_t + \widehat{l}_t^c = \widehat{k}_t^c + \frac{\nu Q^k/PK^c}{W/PL^c}\Big(\widehat{\varrho}_t^k + \nu_t\Big) - \frac{\beta^{-1}(1-\theta^c)Q^k/PK^c}{W/PL^c}\Big(\widehat{\varrho}_{t-1}^k + \widehat{i}_{t-1} - \pi_t\Big) \tag{30}$$

Aggregate inflation dynamics $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ are approximated by:

$$\pi_t \approx \phi\Big(\frac{P^c}{P}\Big)^{1-\varepsilon}\pi_t^c + (1-\phi)\Big(\frac{P^u}{P}\Big)^{1-\varepsilon}\pi_t^u, \quad \pi_t^s \equiv log(\frac{P_t^s}{P_{t-1}^s}),\ s = \overline{u,c}, \tag{31}$$

whereas the goods market clearing condition by:

$$\widehat{y}_t \approx \widehat{c}_t, \qquad y_t \equiv log(Y_t) \text{ with } Y_t \equiv \Big[\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}}di\Big]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{32}$$

the labor market clearing condition by:

$$\widehat{l}_t \approx \phi\frac{L^c}{L}\widehat{l}_t^c + (1-\phi)\frac{L^u}{L}\widehat{l}_t^u, \tag{33}$$

and the one on the capital market by:

$$0 \approx \phi\frac{K^c}{K^u}\widehat{k}_{t+1}^c + (1-\phi)\widehat{k}_{t+1}^u \tag{34}$$

Debt market clears at the nominal interest rate set by the monetary authority(13).

Table 6: Calibration full model specification with sticky wages

| | |
|---|---|
| Core parameters | |
| $\sigma = 1$ | intertemporal elasticity of substitution |
| $\beta = 0.99$ | 4% steady-state real (annualized) rate |
| $\varphi = 5$ | Frisch elasticity of labor supply 0.2 |
| $\alpha = 0.25$ | share of labor in total output 75% |
| $\varepsilon = 9$ | steady-state price markup 12.5% |
| $\xi$ | average $\kappa$ equals Calvo counterpart for $\theta = 0.75$ |
| $\lambda_w$ | match the Calvo counterpart $\theta_w = 3/4$ and $\varepsilon_w = 4.5$ |
| **Credit frictions** | |
| $\phi \in [0, 1]$ | considered a variable in the analysis |
| *Baseline* | |
| $\alpha = 0.25$ | elasticity of output to real-estate (Gali (2015)) |
| $\nu = 0.8$ | pledgeability ratio of real-estate as collateral |
| $\theta^c = 0.23$ | fraction real-estate equity-financed by constrained firms |
| *Alternative 1* | |
| $\alpha = 0.03$ | elasticity of output to real-estate (Iacoviello (2005)) |
| $\nu = 0.8$ | pledgeability ratio of real-estate as collateral |
| $\theta^c = 0.23$ | fraction real-estate equity-financed by constrained firms |
| *Alternative 2* | |
| $\alpha = 0.03$ | elasticity of output to real-estate (Iacoviello (2005)) |
| $\nu = 0.8$ | pledgeability ratio of real-estate as collateral |
| $\theta^c = 0.45$ | fraction real-estate equity-financed by constrained firms |
| **Shock persistence** | |
| $\rho_m = 0$ | transitory monetary impulse |
| $\rho_z = 0.5$ | persistent demand shock |
| $\rho_a = 0.9$ | persistent technology shock |

## 10.2 The efficient allocation (following closely Gali (2015))

The efficient allocation associated with the model economy can be determined by solving the problem facing a benevolent social planner seeking to maximize the representative household's welfare, given technology and preferences. Given the absence of mechanisms for the economy as a whole to transfer resources across periods (e.g. capital accumulation), the efficient allocation corresponds to the solution of a sequence of static social planner problems. Specifically, for each period the optimal allocation must maximize the household's utility:

$$U(C_t, \{L_t(j)\}; e^{z_t}, e^{\chi_t}) \tag{35}$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$ and $L_t \equiv \int_0^1 \frac{L_t(j)^{1+\varphi}}{1+\varphi} dj$ subject to the resource constraints:

$$C_t(i) = A_t H_t(i)^\alpha L_t(i)^{1-\alpha}, \quad \forall i \in [0,1] \tag{36}$$

$$L_t(i) \equiv \left(\int_0^1 L_t(i,j)^{1-\frac{1}{\epsilon_w}} dj\right)^{\frac{\epsilon_w}{\epsilon_w-1}}, \quad \forall i \in [0,1] \tag{37}$$

$$\int_0^1 H_t(i) di = \bar{K} \tag{38}$$

$$\int_0^1 L_t(i) di = L_t \tag{39}$$

The associated optimality conditions are:

$$C_t(i) = C_t, \forall i \in [0,1] \tag{40}$$

$$L_t(i,j) = L_t(j) = L_t(i) = L_t, \forall i \in [0,1], \forall j \in [0,1] \tag{41}$$

$$H_t(i) = \bar{K}, \forall i \in [0,1] \tag{42}$$

$$-\frac{U_{l,t}}{U_{c,t}} = (1-\alpha) A_t \bar{K}^\alpha L_t^{1-\alpha} \tag{43}$$

Thus it is optimal to produce and consume the same quantity of all goods and to allocate the same amount of labor and capital to all firms. That result is a consequence of all goods entering the utility function symmetrically, combined with the concavity of utility and production functions, identical for all goods. Once the symmetric allocation is imposed, the remaining condition defining the efficient allocation, equation (43), equates the marginal rate of substitution between consumption and employment to the corresponding marginal rate of transformation (which in turn equals the marginal product of labor).

The NK model developed in this paper is characterized by three different distorsions: (i) the presence of market power in goods and labor markets exercised by monopolistically competitive firms and labor unions, respectively; (ii) infrequent price and wage adjustment by firms/labor unions; (iii) the presence of credit frictions affecting production of some firms.

In the decentralized economy, in the credit-frictionless limit, in the absence of nominal rigidities, (40), (41) and (42) are satisfied. In this case:

$$\frac{W_t}{P_t} = -\frac{U_{t,t}}{U_{c,t}}\mathscr{M}_w \tag{44}$$

and

$$P_t = \mathscr{M}_p \frac{W_t}{(1-\alpha)A_t\bar{K}^{\alpha}L_t^{1-\alpha}} \tag{45}$$

Note that an employment subsidy $\tau = 1 - \frac{1}{\mathscr{M}_p\mathscr{M}_w}$ funded with lump-sum taxes can be used to guaranteeing the efficiency of the flexible price/flexible wage equilibrium in the credit frictionless limit. Namely, with this subsidy in place:

$$P_t = \mathscr{M}_p \frac{(1-\tau)W_t}{(1-\alpha)A_t\bar{K}^{\alpha}L_t^{1-\alpha}} \tag{46}$$

and hence, condition (43) is also satisfied. In the welfare analysis, I assume such a subsidy is in place. Thus, the flexible-price credit-frictionless specification is the efficient benchmark at all times.

## 10.3 Zero inflation steady-state

It is convenient to first derive the steady-state shadow value of the credit collateral constraint. The model is calibrated such that the associated constraint binds, namely:

$$\left[\nu - \beta^{-1}\Big(1-\theta^c\Big)\right]\frac{Q^k}{P}\frac{K^c}{Y^c} = (1-\tau)\frac{W}{P}\frac{L^c}{Y^c} \tag{47}$$

Using the price-setting equation of the constrained group of firms:

$$\frac{P^c}{P} = \mathscr{M}_p(1-\tau)\frac{W}{P}\frac{L^c}{Y^c}\frac{1+\lambda^c}{1-\alpha} \Rightarrow (1-\tau)\frac{W}{P}\frac{L^c}{Y^c} = \mathscr{M}_p^{-1}\frac{P^c}{P}\frac{1-\alpha}{1+\lambda^c} \tag{48}$$

and their real estate demand equation:

$$\frac{Q^k}{P} = \frac{\alpha\frac{Y^c}{K^c}\frac{P^c}{P}}{\left[\beta^{-1} - 1 - \lambda^c\Big(\nu - \beta^{-1}(1-\theta^c)\Big)\right]} \tag{49}$$

we can derive the equilibrium expression of $\lambda^c$ exclusively as a function of structural parameters:

$$\lambda^c = \frac{(1-\alpha)(\beta^{-1}-1)}{\mathscr{M}_p\left[\nu - \beta^{-1}\Big(1-\theta^c\Big)\right]} - \alpha \tag{50}$$

Next, it is convenient to compute the steady-state value $\delta^p \equiv \frac{P^c}{P^u}$. Using the production functions for the two groups of firms we obtain:

$$\frac{Y^c}{Y^u} = \Big(\frac{K^c}{K^u}\Big)^{\alpha}\Big(\frac{L^c}{L^u}\Big)^{1-\alpha} \tag{51}$$

Furthermore, the real estate market equilibrium $\frac{MRK^c}{P} = \frac{MRK^u}{P}$,

$$\frac{\alpha\frac{Y^u}{K^u}\frac{P^u}{P}}{\beta^{-1}-1} = \frac{\alpha\frac{Y^c}{K^c}\frac{P^c}{P}}{\Big[\beta^{-1}-1-\lambda^c\Big(\nu-\beta^{-1}(1-\theta^c)\Big)\Big]} \tag{52}$$

implies:

$$\frac{K^c}{K^u} = \frac{Y^c}{Y^u}\delta^p\delta, \quad \delta \equiv \frac{\beta^{-1}-1}{\Big[\beta^{-1}-1-\lambda^c\Big(\nu-\beta^{-1}(1-\theta^c)\Big)\Big]} \tag{53}$$

Using $Y^u = \Big(\frac{P^u}{P}\Big)^{-\varepsilon}Y$ and $Y^c = \Big(\frac{P^c}{P}\Big)^{-\varepsilon}Y$, we can determine:

$$\frac{Y^c}{Y^u} = \Big(\frac{P^c}{P^u}\Big)^{-\varepsilon} = (\delta^p)^{-\varepsilon} \tag{54}$$

and write (53) as:

$$\frac{K^c}{K^u} = \Big(\delta^p\Big)^{1-\varepsilon}\delta \tag{55}$$

Replacing (55) and (54) in (51), it yields:

$$\frac{L^c}{L^u} = \Big(\delta^p\Big)^{-\frac{\varepsilon+x(1-\varepsilon)}{1-\alpha}}\Big(\delta\Big)^{-\frac{x}{1-\alpha}} \tag{56}$$

One way to determine $\delta^p$ is to express the tight collateral constraint (47) in terms of the labor-output ratio of unconstrained firms in equilibrium. The expression of $L^u/Y^u$ as a function of $L^c/Y^c$ can be determined by replacing the expression of $K^c/K^u$ from (53) in (51), namely:

$$\frac{Y^c}{Y^u} = \Big(\frac{Y^c}{Y^u}(\delta^p)\delta\Big)^{\alpha}\Big(\frac{L^c}{L^u}\Big)^{1-\alpha} \Rightarrow \frac{L^c/Y^c}{L^u/Y^u} = \Big(\delta^p\delta\Big)^{-\frac{\alpha}{1-\alpha}} \tag{57}$$

After replacing in the binding credit collateral constraint (47) the expressions of $L^c/Y^c$ from (57) and of $\frac{Q^k}{P}\frac{K^c}{Y^c}$ from: (49)

$$\frac{\Big(\nu-\beta^{-1}(1-\theta^c)\Big)\alpha\frac{P^c}{P}}{\Big[\beta^{-1}-1-\lambda^c\Big(\nu-\beta^{-1}(1-\theta^c)\Big)\Big]} = (1-\tau)\frac{W}{P}\frac{L^u}{Y^u}\Big(\delta^p\delta\Big)^{-\frac{\alpha}{1-\alpha}} \tag{58}$$

and using the price-setting equation of unconstrained firms:

$$\frac{P^u}{P} = \mathscr{M}_p \frac{1-\tau}{1-\alpha}\frac{W}{P}\frac{L^u}{Y^u} \Rightarrow (1-\tau)\frac{W}{P}\frac{L^u}{Y^u} = \mathscr{M}_p^{-1}(1-\alpha)\frac{P^u}{P} \tag{59}$$

we can compute the expression of $\delta^p$ as a function of structural parameters:

$$\delta^p = \left[\frac{1-\alpha}{\alpha}\frac{\beta^{-1} - 1 - \lambda^c\Big(\nu - \beta^{-1}(1-\theta^c)\Big)}{\Big(\nu - \beta^{-1}(1-\theta^c)\Big)\mathscr{M}_p}\right]^{1-\alpha}\delta^{-\alpha} \tag{60}$$

The value of $\delta^p$ can be directly used in (55) to determine $\frac{K^c}{K^u} = (\delta^p)^{1-\varepsilon}\delta$. Furthermore, using the labor market clearing condition:

$$L = \phi L^c + (1-\phi)L^u \Rightarrow 1 = \phi\frac{L^c}{L} + (1-\phi)\frac{L^u}{L} \tag{61}$$

and the expression of $L^c$ as a function of $L^u$ in (56), it yields:

$$\frac{L^u}{L} = \left(\phi\Big(\delta^p\Big)^{-\frac{\varepsilon+\alpha(1-\varepsilon)}{1-\alpha}}\Big(\delta\Big)^{-\frac{\alpha}{1-\alpha}} + (1-\phi)\right)^{-1} \tag{62}$$

and $\frac{L^c}{L} = \frac{L^u}{L}\Big(\delta^p\Big)^{-\frac{\varepsilon+\alpha(1-\varepsilon)}{1-\alpha}}\Big(\delta\Big)^{-\frac{\alpha}{1-\alpha}}$. The steady-state expression of the price index $P^{1-\varepsilon} = \phi(P^c)^{1-\varepsilon} + (1-\phi)(P^u)^{1-\varepsilon}$ implies:

$$1 = \phi\left(\frac{P^c}{P}\right)^{1-\varepsilon} + (1-\phi)\left(\frac{P^u}{P}\right)^{1-\varepsilon} \tag{63}$$

Using $\delta^p = \frac{P^c/P}{P^u/P}$, $\frac{P^u}{P}$ can be determined as a function of structural parameters as:

$$1 = \phi\left(\delta^p\frac{P^u}{P}\right)^{1-\varepsilon} + (1-\phi)\left(\frac{P^u}{P}\right)^{1-\varepsilon} \Rightarrow \frac{P^u}{P} = \left(\phi\Big(\delta^p\Big)^{1-\varepsilon} + (1-\phi)\right)^{\frac{1}{\varepsilon-1}} \tag{64}$$

and the one of $\frac{P^c}{P} = \frac{P^u}{P}\delta^p$.

In the analysis of monetary policy transmission, there is no employment subsidy correcting for market power distorsions ($\tau = 0$), whereas in the welfare analysis this subsidy is assumed to be in place.

## 10.4 Plots monetary policy transmission

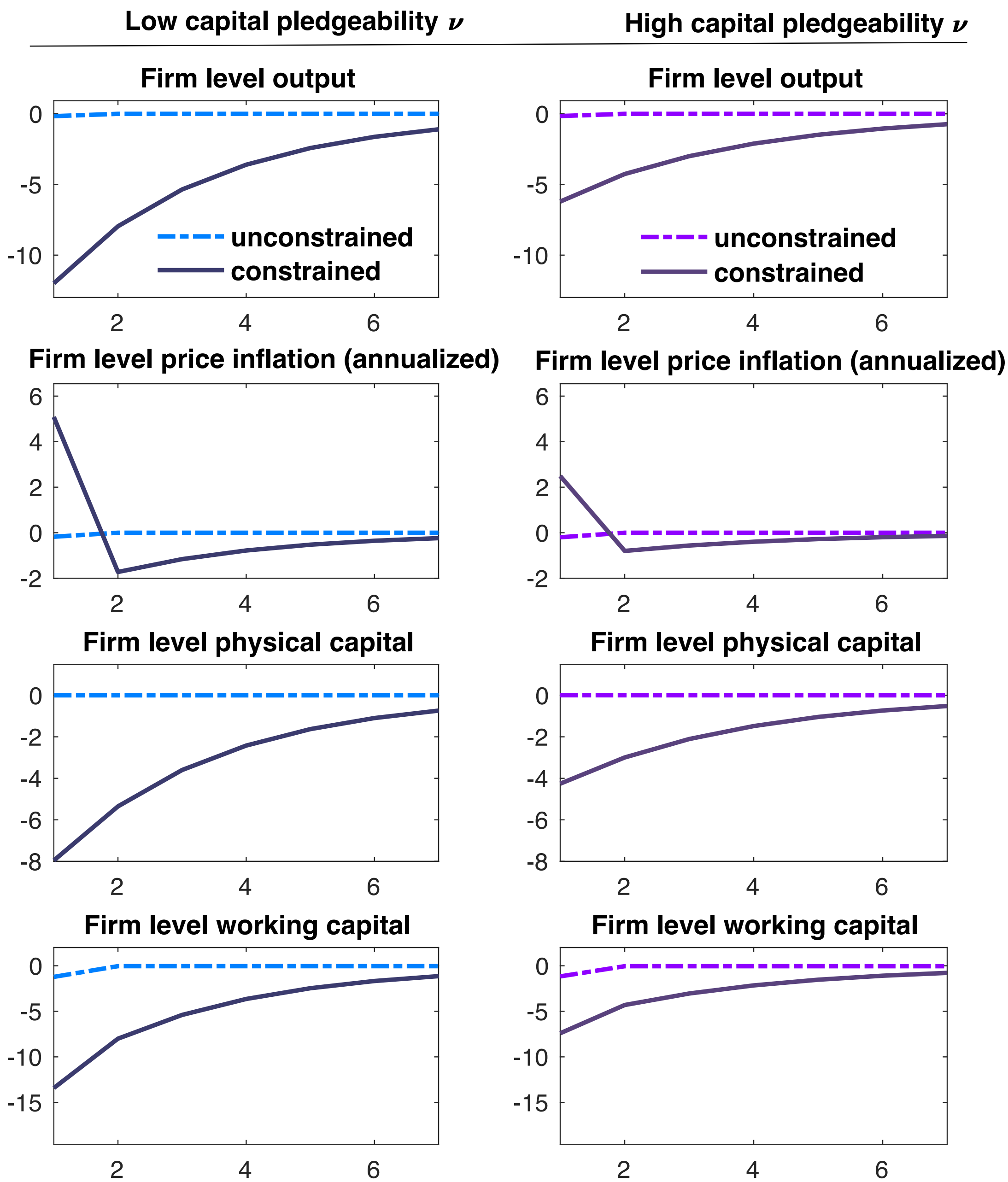


Figure 27: Dynamic response to a transitory monetary tightening ($\alpha = 0.03$)

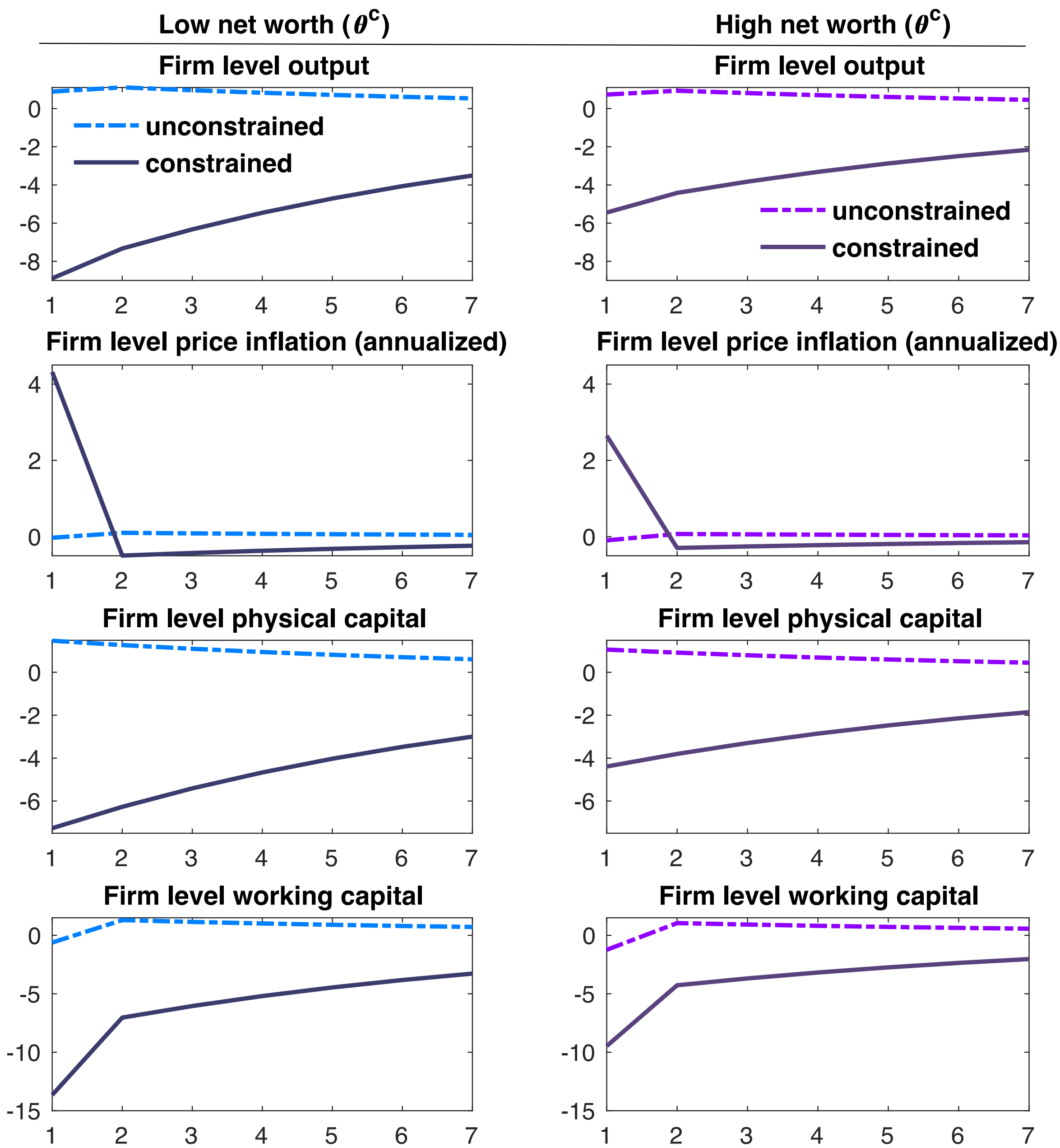


Figure 28: Dynamic response to a transitory monetary tightening

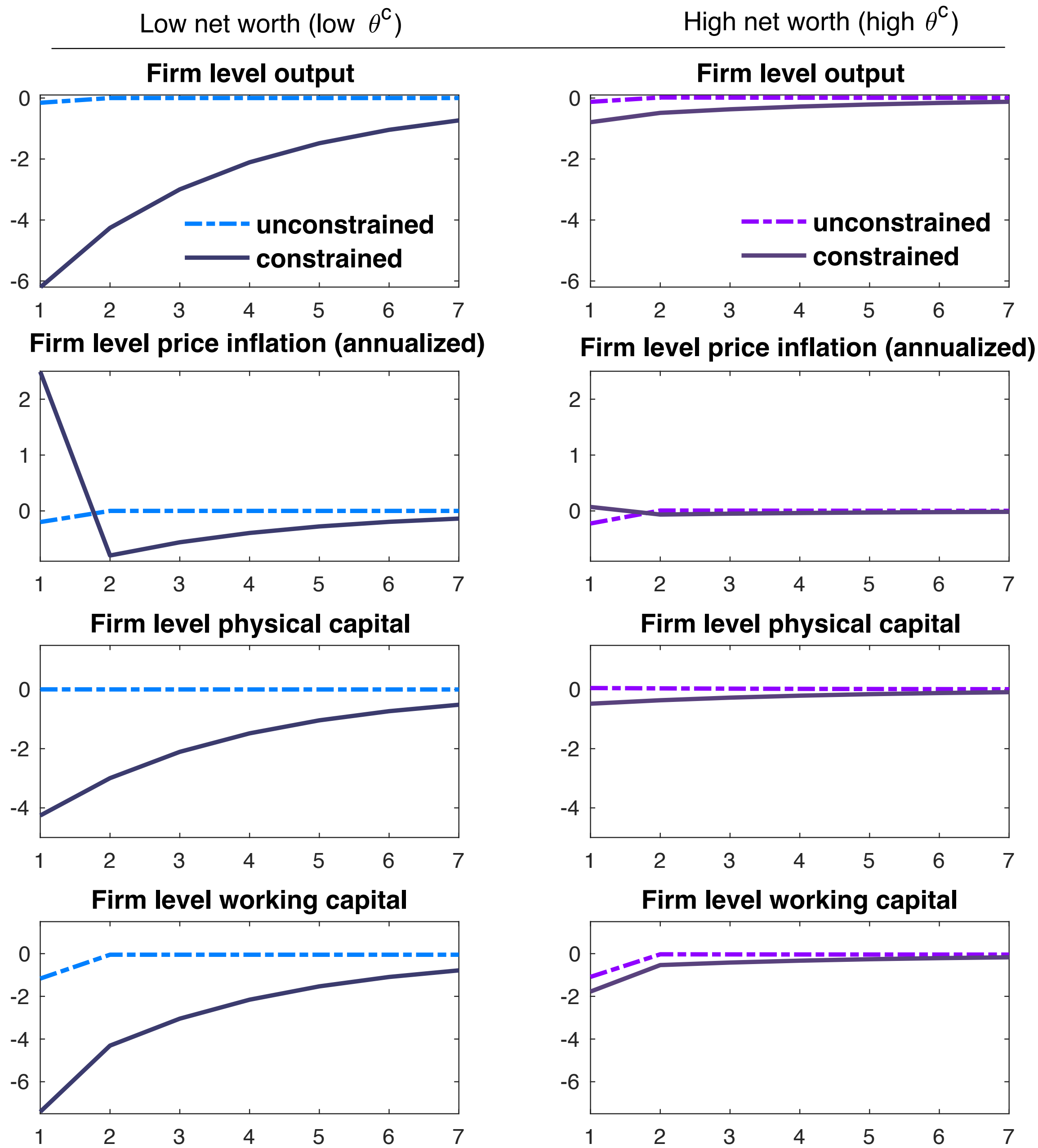


Figure 29: Dynamic response to a transitory monetary tightening ($\alpha = 0.03$)

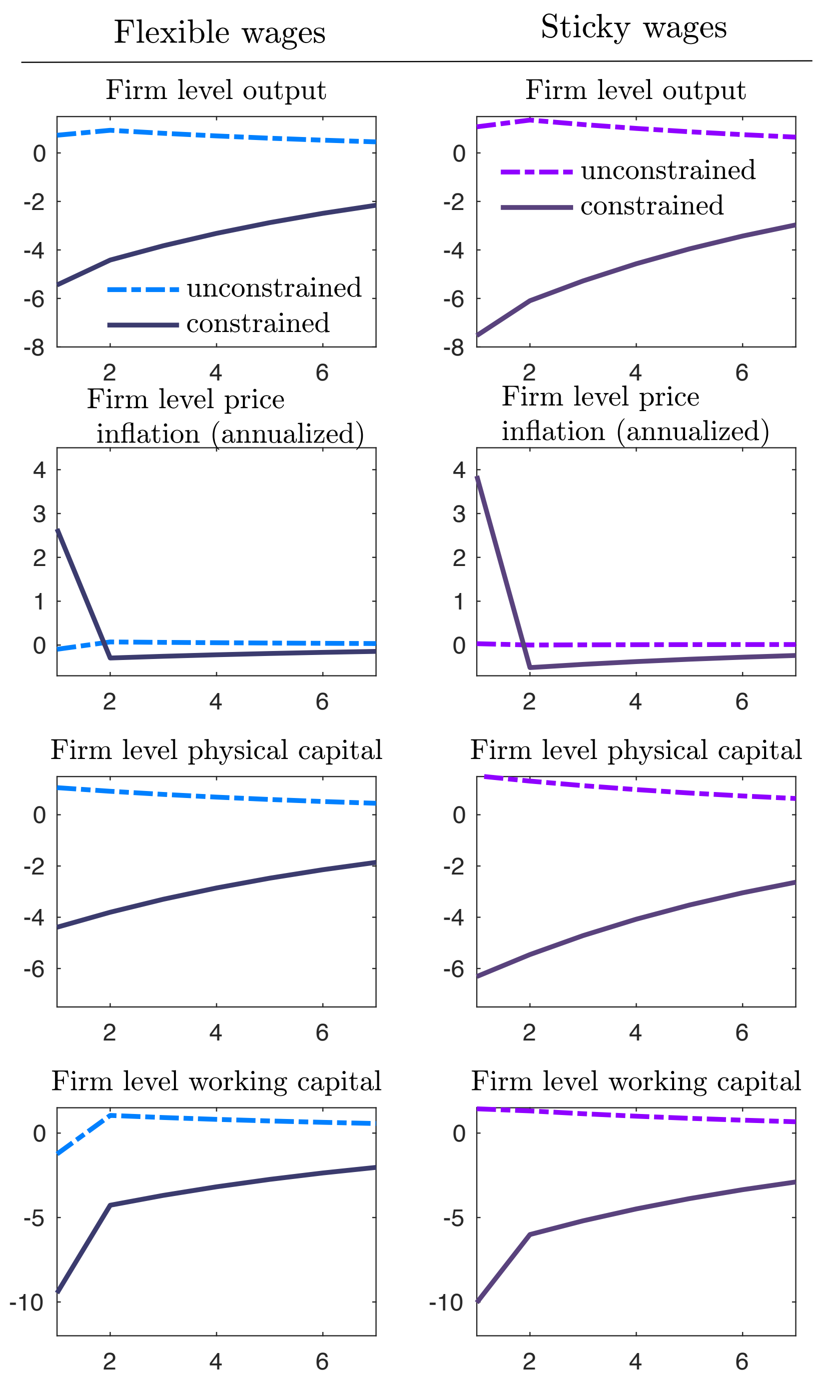


Figure 30: Dynamic response to a transitory monetary tightening

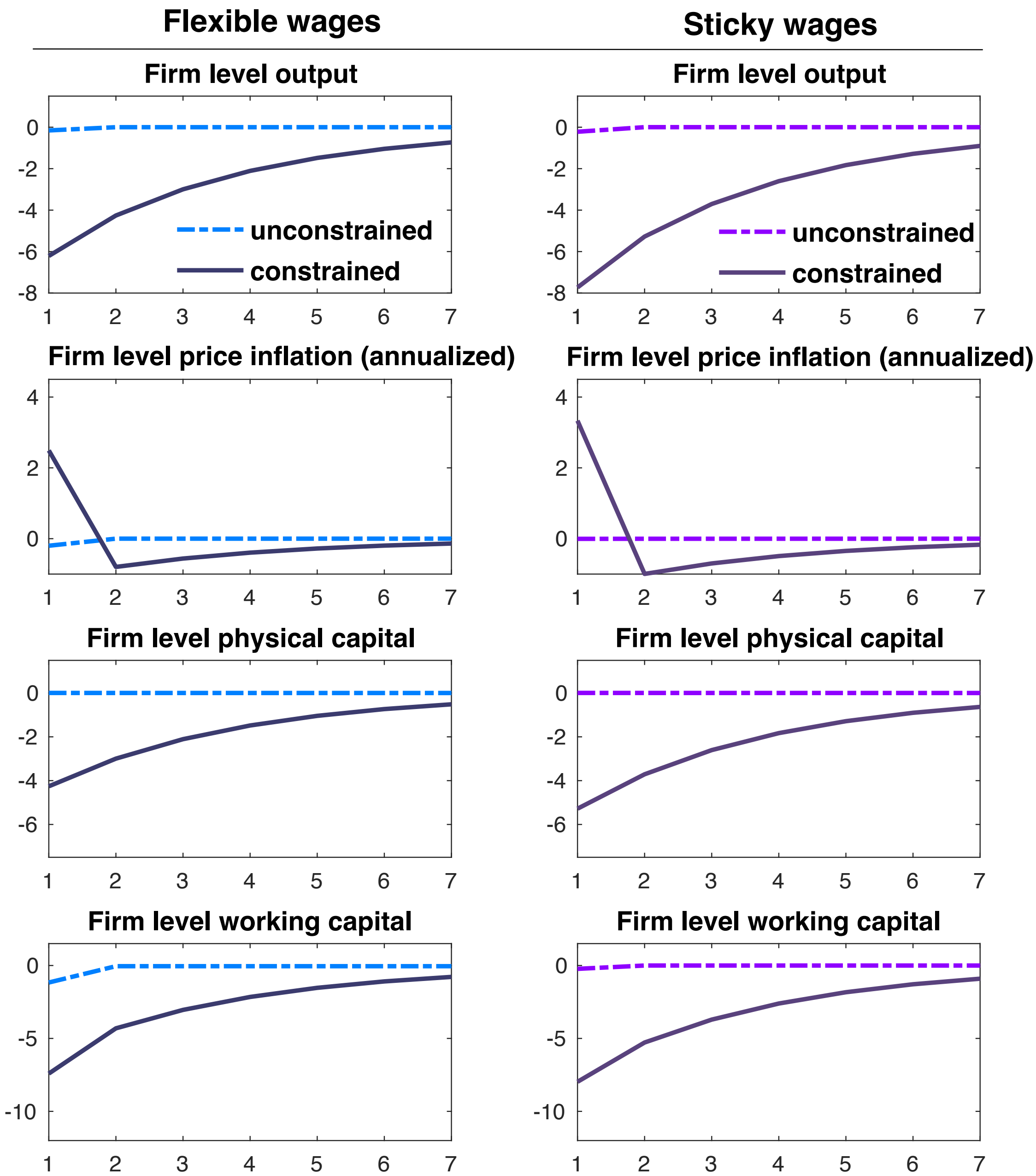


Figure 31: Dynamic response to a transitory monetary tightening ($\alpha = 0.03$)

### 10.5 Welfare loss function

Period utility can be written up to a second order approximation as:

$$\frac{U_t - U}{U_c Y} \approx (1+z_t)\widehat{c}_t + \frac{1-\sigma}{2}\widehat{c}_t^2 + \frac{U_l L}{U_c Y}\left((1+z_t)\int_0^1 \widehat{l}_t(j)dj + \frac{1+\varphi}{2}\int_0^1 \widehat{l}_t^2(j)dj\right) + t.i.p. \tag{65}$$

Next, we express $\int_0^1 \widehat{l}_t(j)dj$ and $\int_0^1 \widehat{l}_t^2(j)dj$ as a function of aggregate working hours. We define the aggregate labor hours (of all types $j$) used by all firms in the economy:

$$L_t \equiv \int_0^1 L_t(j)dj \tag{66}$$

Up to a second-order approximation, the following relations hold (Gali (2015), p. 189):

$$\int_0^1 \widehat{l}_t^2(j)dj \approx \widehat{l}_t^2 + \varepsilon_w^2 var_j\{w_t(j)\}$$
$$\int_0^1 \widehat{l}_t(j)dj \approx \widehat{l}_t - \frac{1}{2}\varepsilon_w^2 var_j\{w_t(j)\}$$

Replacing these expressions in (65), we obtain:

$$\begin{aligned}\frac{U_t - U}{U_c Y} \approx& (1+z_t)\widehat{c}_t + \frac{1-\sigma}{2}\widehat{c}_t^2 + \frac{U_l L}{U_c Y}\Big[(1+z_t)\Big(\widehat{l}_t - \frac{1}{2}\varepsilon_w^2 var_j\{w_t(j)\}\Big) + \\ &+ \frac{1+\varphi}{2}\Big(\widehat{l}_t^2 + \varepsilon_w^2 var_j\{w_t(j)\}\Big)\Big] + t.i.p. \\ \approx& (1+z_t)\widehat{c}_t + \frac{1-\sigma}{2}\widehat{c}_t^2 + \frac{U_l L}{U_c Y}\Big[(1+z_t)\widehat{l}_t + \frac{1+\varphi}{2}\widehat{l}_t^2 + \\ &+ \frac{\varphi\varepsilon_w^2}{2}\varepsilon_w^2 var_j\{w_t(j)\}\Big] + t.i.p.\end{aligned}$$

Next we express $\widehat{c}_t$ as a function of $\widehat{y}_t$. Up to a second order approximation the goods market clearing condition writes:

$$\widehat{c}_t \approx \widehat{y}_t - \frac{1}{2}\Big[(1-\phi)\xi(\pi_t^u)^2 + \phi\xi(\pi_t^c)^2\Big]$$

$$\text{Thus, } \frac{U_t - U}{U_c Y} \approx (1+z_t)\widehat{y}_t - \frac{1}{2}\Big[(1-\phi)\xi(\pi_t^u)^2 + \phi\xi(\pi_t^c)^2\Big] + \frac{1-\sigma}{2}\widehat{y}_t^2 + \frac{U_l L}{U_c Y}\Big[(1+z_t)\widehat{l}_t + \frac{1+\varphi}{2}\widehat{l}_t^2 + \frac{\varphi\varepsilon_w^2}{2}\varepsilon_w^2 var_j\{w_t(j)\}\Big] + t.i.p. \tag{67}$$

Next, we write $\widehat{y}_t + \frac{U_l L}{U_c Y}\widehat{l}_t$ as a function of quadratic terms[42]. $L_t$ in (66) equals:

$$L_t = \int_0^1 \int_0^1 L_t(i,j) dj di = \Delta_{w,t}\Big[\phi L_t^c + (1-\phi) L_t^u\Big], \ \ \Delta_{w,t} \equiv \int_0^1 \Big(\frac{W_t(j)}{W_t}\Big)^{-\varepsilon_w} dj$$

Up to a second order approximation this relation implies:

$$\widehat{l}_t \approx \frac{\varepsilon_w}{2} var_j\{w_t(j)\} + \phi \frac{L^c}{L}\Big[\widehat{l}_t^c + \frac{1}{2}(\widehat{l}_t^c)^2\Big] + (1-\phi)\frac{L^u}{L}\Big[\widehat{l}_t^u + \frac{1}{2}(\widehat{l}_t^u)^2\Big]$$

where $var_j\{w_t(j)\}$ defines the second order approximation of $\int_0^1 (W_t(j) - W_t)^2 dj$. Using the expression of aggregate output, the production functions, and the expression above, $\widehat{y}_t + \frac{U_l L}{U_c Y}\widehat{l}_t$ equals:

$$\begin{aligned}
\widehat{y}_t + \frac{U_l L}{U_c Y}\widehat{l}_t =& \phi\Big(\frac{P^c}{P}\Big)^{1-\varepsilon}\widehat{y}_t^c + (1-\phi)\Big(\frac{P^u}{P}\Big)^{1-\varepsilon}\widehat{y}_t^u + \\
&+ \frac{U_l L}{U_c Y}\Big[\frac{\varepsilon_w}{2} var_j\{w_t(j)\} + \phi\frac{1}{2}\frac{L^c}{L}(\widehat{l}_t^c)^2 + \frac{1}{2}(1-\phi)\frac{L^u}{L}(\widehat{l}_t^u)^2 + \\
&+ \phi\frac{L^c}{L}\frac{\widehat{y}_t^c - a_t - \alpha\widehat{k}_t^c}{1-\alpha} + (1-\phi)\frac{L^u}{L}\frac{\widehat{y}_t^u - a_t - \alpha\widehat{k}_t^u}{1-\alpha}\Big] + t.i.p.
\end{aligned}$$

Replacing the expression above in (67), we get (after some algebra),

$$\begin{aligned}
\frac{U_t - U}{U_c Y} \approx& -\frac{1}{2}\Big[\xi(1-\phi)(\pi_t^u)^2 + \xi\phi(\pi_t^c)^2 - (1-\sigma)\widehat{y}_t^2 + \\
&+ \frac{(1-\alpha)(1+\varphi)}{L^u/L}\Big(\frac{P^u}{P}\Big)^{1-e}\widehat{l}_t^2 + \\
&+ \frac{(1-\alpha)e_w(1+\varphi e_w)}{L^u/L}\Big(\frac{P^u}{P}\Big)^{1-e} var_j\{w_t(j)\} + \\
&+ \frac{(1-\alpha)}{L^u/L}\Big(\frac{P^u}{P}\Big)^{1-\varepsilon}\Big[\phi\frac{L^c}{L}(\widehat{l}_t^c)^2 + (1-\phi)\frac{L^u}{L}(\widehat{l}_t^u)^2\Big] - \\
&- 2\phi(\frac{P^c}{P})^{1-\epsilon}\frac{\lambda^c}{1+\lambda^c}\widehat{y}_t^c - 2\alpha\phi\Big((\frac{P^c}{P})^{1-\epsilon}\frac{1}{1+\lambda^c} - (\frac{P^u}{P})^{1-\epsilon}\frac{K^c}{K^u}\Big)\widehat{k}_t^c \\
&+ \alpha(\frac{P^u}{P})^{1-\epsilon}\Big(\frac{K^c}{K^u}\phi(\widehat{k}_t^c)^2 + (1-\phi)(\widehat{k}_t^u)^2\Big)\Big] + t.i.p.
\end{aligned}$$

---

[42] I tried first to express $\widehat{l}_t$ as a function of aggregate output. In the basic credit-frictionless NK model $L_t = \Delta_{w,t}\Delta_{p,t}\Big(\frac{Y_t}{A_t}\Big)^{\frac{1}{1-\alpha}}$ with $\Delta_{w,t} \equiv \int_0^1 \Big(\frac{W_t(j)}{W_t}\Big)^{-\varepsilon_w} dj$, $\Delta_{p,t} \equiv \int_0^1 \Big(\frac{P_t(i)}{P_t}\Big)^{\frac{-\varepsilon_p}{1-\alpha}} di$. Hence, up to a second order approximation, $(1-\alpha)\widehat{l}_t = \widehat{y}_t - a_t + \frac{(1-\alpha)\varepsilon_w}{2} varj\{w_t(j)\} + \frac{\varepsilon_p}{2\Theta} vari\{p_t(i)\}$ (Gali (2015), Chapter 6, page 190). Here, due to the presence of capital in the production function, and of the different capital levels used by unconstrained versus constrained firms, $L_t = \Delta_{w,t}\Delta_{pk,t}\Big(\frac{Y_t}{A_t}\Big)^{\frac{1}{1-\alpha}}$, with $\Delta_{pk,t} \equiv \int_0^1 \Big(\frac{P_t(i)}{P_t}\Big)^{\frac{-\varepsilon_p}{1-\alpha}} K_t(i)^{-\frac{\alpha}{1-\alpha}} di$. Thus, in the second order approximation of the expression of aggregate labor there are first and second order terms including $\widehat{k}_t^u$ and $\widehat{k}_t^c$. The approach followed is less computational intensive.

We can thus define the loss function as:

$$
\begin{aligned}
\mathscr{L}_0 \approx & -E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c Y} \\
& = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \Big[ \xi(1-\phi)(\pi_t^u)^2 + \xi\phi(\pi_t^c)^2 - (1-\sigma)\widehat{y}_t^2 + \gamma^l \widehat{l}_t^2 + \gamma^w (\widehat{\pi}_t^w)^2 + \\
& + \gamma^{lcu}\Big(\phi \frac{L^c}{L}(\widehat{l}_t^c)^2 + (1-\phi)\frac{L^u}{L}(\widehat{l}_t^u)^2\Big) - \gamma^{yc}\widehat{y}_t^c - \gamma^{kc}\widehat{k}_t^c + \\
& + \gamma^{kcu}\Big(\frac{K^c}{K^u}\phi(\widehat{k}_t^c)^2 + (1-\phi)(\widehat{k}_t^u)^2\Big)\Big] + t.i.p.
\end{aligned}
$$

where I used $\sum_{t=0}^{\infty} \beta^t var_j\{w_t(j)\} = \frac{\theta^w}{(1-\beta\theta^w)(1-\theta^w)}(\pi_t^w)^2$ (see Gal015), page 119): $\gamma^l \equiv \frac{(1-\alpha)(1+\varphi)}{L^u/L}(\frac{P^u}{P})^{1-\epsilon} \geq 0$,
$\gamma^w \equiv \frac{(1-\alpha)\epsilon^w(1+\varphi\epsilon^w)\theta^w}{L^u/L(1-\beta\theta^w)(1-\theta^w)}(\frac{P^u}{P})^{1-\epsilon} \geq 0$,
$\gamma^{lcu} \equiv \frac{(1-\alpha)}{L^u/L}(\frac{P^u}{P})^{1-\epsilon} \geq 0$,
$\gamma^{yc} \equiv 2\phi(\frac{P^c}{P})^{1-\epsilon}\frac{\lambda^c}{1+\lambda^c} \geq 0$,
$\gamma^{kc} \equiv 2\alpha\phi\Big((\frac{P^c}{P})^{1-\epsilon}\frac{1}{1+\lambda^c} - (\frac{P^u}{P})^{1-\epsilon}\frac{K^c}{K^u}\Big) \geq 0$,
$\gamma^{kcu} \equiv \alpha(\frac{P^u}{P})^{1-\epsilon} \geq 0$,
$t.i.p.$ collects terms that are independent of monetary policy. I derive the optimal monetary policy under commitment by choosing all variables in section to minimize the welfare loss function above subject to the equations of the model in section 10.1.

Table 7: Coefficients linear terms welfare criterion

| | $\phi = 0$ | $\phi = 0.2$ | $\phi = 0.5$ |
|---|---|---|---|
| *Baseline* | | | |
| $\gamma^{yc}$ | 0 | 0.017 | 0.046 |
| $\gamma^{kc}$ | 0 | -0.015 | -0.041 |
| *Alternative 2* | | | |
| $\gamma^{yc}$ | 0 | 0.0019 | 0.0051 |
| $\gamma^{kc}$ | 0 | $-0.0017$ | $-0.0045$ |

## 10.6 Plots optimal monetary policy

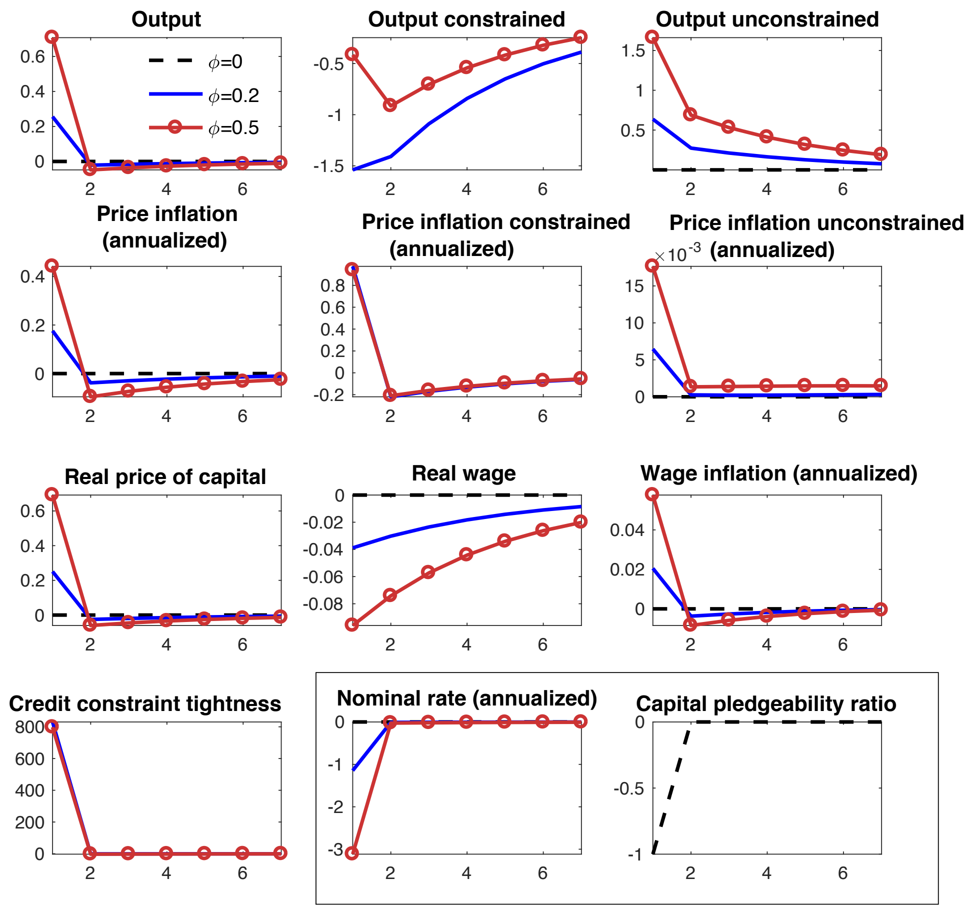


Figure 32: Optimal policy: Dynamic responses to a transitory negative pledgeability ratio shock for different shares of constrained firms ($\alpha = 0.03$, $\theta^c = 0.45$)