Inside-Money in the New Keynesian Model

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Abstract

The textbook New Keynesian framework has become a common tool for monetary policy analysis in central banks. Policymakers are nonetheless often concerned that this framework abstracts away from endogenous money creation, and lacks realism. To address this concern, I introduce endogenous money creation by the private banking sector (like deposits), or "inside money", into the textbook framework. I find that the new "inside money" model has the same equilibrium representation as the textbook "money-less" one, and hence transmission and optimal design of monetary policy in the two models are identical.

Keywords: New Keynesian model, inside-money, cashless, inside-liquidity banking theory

JEL Class.: E2 – E3 – E4

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1 Introduction

A number of economists have expressed concerns over the lack of explicit account of banks' monetary role in New Keynesian (NK) models widely used for monetary policy analysis. These concerns have been expressed on two dimensions. On a first dimension, they regard the monetary fundamentals of cashless versions of these models, because they do not explicitly model the role of bank deposits ("inside-money") in transactions. For instance, Goodfriend and McCallum (2007) consider the NK framework in Bernanke, Gertler and Gilchrist (1999) as "fundamentally non-monetary" because "it does not recognize the existence of a demand for money that serves to facilitate transactions". Similarly, Borio and Disyatat (2011) and Borio (2014) fear that model economies in such frameworks may fundamentally represent real (barter) ones, and that one may need better analytical representations of actual monetary economies.

On a second dimension, the concerns over the lack of explicit reference to the role of banks in money creation are related to the way the behaviour of banks is modeled in these setups (e.g. Ryan-Collins et al. (2011), Borio and Disyatat (2011), Borio (2014), Jakab and Kumhof (2015), Turner (2016)). Specifically, banks are assumed to exclusively channel (real) resources from "savers" to "debtors"[1]. This hypothesis is however *a priori* at odds with the functioning of banking systems in practice where banks extend loans by issuing deposits ("inside-money")[2].

I address these two related concerns in this paper. Regarding the first one, I find that the "cashless" basic NK setup (e.g. Woodford (2003), Galí (2015)) is isomorphic to monetary versions where bank deposits ("inside-money") issued within a perfectly competitive banking sector are used in transactions. Hence, this basic model is not inconsistent with the equilibrium dynamics of a monetary economy with inside money. Furthermore, to address the second dimension, I show that accounting explicitly for banks' monetary role in the canonical model of Bernanke, Gertler and Gilchrist (1999) is irrelevant for its equilibrium dynamics, namely that a version with banks extending loans by issuing deposits (in line with practice) is isomorphic to their original specification. These results do not imply however that banks' monetary role is generally irrelevant for monetary policy analyses within the NK paradigm. To make this point, I end with a number of research questions for which accounting for this role is essential.

To give a hint on the first result, note that according to the "cashless" definition in Woodford (1998), cashless models are meant to describe "pure-credit economies" with central bank liabilities playing the role of unit of account. Particularly, they are not meant to describe neither barter, nor monetary economies, but a third distinct category[3]. Specifically, according to Woodford (1998), the

[1] For instance, from households to entrepreneurs for investment purposes (e.g. Bernanke, Gertler and Gilchrist (1999)), or from patient households to impatient ones for consumption purposes (e.g. Curdia and Woodford (2016)).

[2] Detailed descriptions of the role of inside-money in modern payment systems are provided for example in publications by the Bank for International Settlements such as Committee on Payment and Settlement Systems (2003), Disyatat (2008) or Borio and Disyatat (2011).

[3] *Barter* is a simultaneous exchange of commodities, whether goods or labor services, with bargaining and without using money (The new Palgrave dictionary of economics (Vol. 1, pp. 384)) whereas in a *monetary economy* the medium of exchange is money (e.g. Collins Dictionary of Economics).

cashless setup describes a world where the execution of trade is decentralised and goods are diversified as in Lucas (1980), but where (nonbank) agents settle transactions on credit by issuing perfectly enforceable IOUs with all payments being carried out *via* book-keeping movements. Otherwise stated, the cashless economy is a hypothetical economy where there are no monetary frictions to justify the use in *transactions* of a distinct perfectly liquid asset such as "money" (Woodford (2003) p.31). Central bank liabilities ("outside-money") do however play a role as a "unit of account" in these setups. Specifically, the monetary authority sets the unit of account in terms of which prices (of both goods and financial assets) are quoted and controls the price level in the economy by setting the price of a one nominal unit of credit in the economy (by issuing a one period nominal bond).

Even though the mapping may not be explicit at first sight, the assumptions of these models are consistent with the role of central banks and the way transactions are settled nowadays in advanced economies. Specifically, central bank liabilities do play the role of unit of account[4]. Furthermore, even though (generally) non-financial agents cannot issue their own IOUs due to a lack of (multilateral) repayment commitment, bank deposits ("inside-money") play in practice the same role as the (underlying) IOUs issued in trade relations in the frictionless cashless version. Intuitively, in line with the actual functioning of banking systems, we may think of banks as being endowed with a repayment enforcement technology, and thus with the ability to exchange nonbank unenforceable IOUs ("bank loans") with their own enforceable IOUs ("bank deposits")[5].

The second result is more elaborate, but, as we will see, inherently linked to the first one. Hereafter, the chapter is organised as follows: Section 2 highlights some of the related theoretical literature; Section 3 describes the analytical setups and derives the two equivalence results; Section 4 discusses a number of research topics whose study within this framework requires an explicit account of banks' monetary role, whereas Section 5 concludes.

2 Contributions to the literature

The present analysis contributes to three main strands of literature. First, it lays out an extension of the NK paradigm. Particularly, it introduces for the first time in this setup bank deposits ("inside-money") created within a banking sector modeled in line with the inside-liquidity theory proposed by Kiyotaki and Moore (2002) and it establishes an isomorphy to its cashless version. Most extensions of the NK model with "money" used in transactions identify it with non-interest bearing central bank liabilities (i.e. banknotes/coins)[6]. In few other NK extensions bank deposits are used in transactions (e.g. Stracca (2007), Goodfriend and McCallum (2008)), but banks' behaviour is not

[4]It is true that in practice central bank liabilities are also used as (outside-)money in transactions. It has been however shown that incorporating outside-money within the New-Keynesian setup has little quantitative significance (Woodford (2003), Ireland (2004), Woodford (2008)). The current analysis takes these results as given and focuses exclusively on the critique outlined in the introduction which concerns the lack of explicit reference to inside-money in such frameworks.

[5]For a description of the functioning of the banking sector in practice, see for instance Disyatat (2008), McLay et al. (2014 a-b), Werner (2014 a-b), Deutsche Bundesbank (2017).

[6]See for instance the derivation of the cashless limit in Woodford (1998).

modelled in line with the inside-liquidity theory and these models don't feature the same neutrality result (in the absence of credit frictions and banks' operational costs)[7].

Second, the paper contributes to the cash-in-advance literature. Particularly, it shows that well-known monetary policy transmission channels such as the inflation/interest rate tax or the cost-channel which emerge when agents need to hold "cash" (i.e. liquid assets) in advance to pay for goods as in Lucas and Stokey (1987), and, respectively, when firms need to finance working capital before receiving proceeds on the sale of output as in Christiano and Eichenbaum (1992), vanish when (interest bearing) inside-money is used instead of (non-interest bearing) outside-money. Moreover, in the absence of nominal rigidities, the Friedman rule (zero nominal interest rates) no longer characterizes optimal monetary policy.

Third, the paper discusses the implementation of monetary policy via the interbank market in the context of the inside-liquidity banking theory developed by Kiyotaki and Moore (2002). This banking theory was developed within a "real" heterogenous agent macroeconomic framework without any reference to monetary policy.

3 The model

The analytical setup is based on the limit case of the monetary economy in Woodford (1998) where agents *do not extend any trade credit* among themselves due to a lack of trust. The setup is adjusted to be interpreted as a standard NK setup (as described for instance in Galí (2015) or Woodford (2003)) enriched with a liquidity(cash)-in-advance constraint. Nominal rigidities are abstracted away without any loss of generality.

To motivate the need for any monetary arrangement, as in Woodford (1998) which follows Lucas (1980, 1981), production and goods' exchanges are carried out in a "decentralised" fashion. Namely, firms are spatially scattered with workers selling labor to a particular firm (producing a particular variety) and consumers obliged to go to the location of each firm to buy the differentiated array of goods. At the beginning of each period, after shocks realise, markets open and equilibrium prices and quantities are determined. Goods' differentiation is a necessary condition for intra-period trade and the use of "money". If all goods were identical, each household would just consume the goods received as counterpart of its labor effort ("wage") and hence no exchange would take place ("autarchy"). However, since each worker wants to consume not only the good produced by the firm employing him, but instead a diversified basket of goods, trade emerges.

Barter would be one possible option as each worker may exchange the type of good produced at his firm with the goods received by other workers. As in Woodford (1998), I implicitly assume

[7] Disyatat (2011) models banks' behaviour in line with this theory (even though it does not explicitly refer to it), but the analytical setup is a partial equilibrium model used to study the implications of modelling banks' behaviour in this way (as opposed to the standard approach) for the bank lending channel of monetary policy transmission.

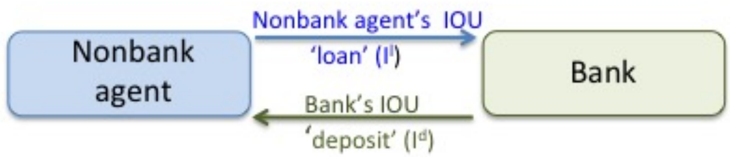

Figure 1: Banks' role in the model in line with inside-liquidity theory

however that this option would entail large (unmodelled) costs making it unappealing in equilibrium[8]. Alternatively, I can assume that all exchanges must be made by the means of a perfectly multilaterally enforceable (liquid) asset. This role is played by central bank liabilities (banknotes, coins), namely "outside money" in standard NK models with monetary frictions. In the current analysis however, I consider the case of "inside-money". Specifically, I assume that even though households and firms cannot directly issue perfectly multilaterally enforceable IOUs, they can exchange them with the multilateral enforceable IOUs of private (trustworthy) third-party agents. The real counterpart of these third-party agents are "banks" and their functioning is modeled in line with inside-liquidity banking theory in Kiyotaki and Moore (2002).

The model economy is thus populated by a continuum of identical households, a continuum of (diversified) monopolistic firms, a perfectly competitive banking sector and a central bank. Banks and firms are owned by households.

3.1 Banks as inside-money suppliers

The behaviour of banks is modeled as in Kiyotaki and Moore (2002). Specifically, as already mentioned, the existence of banks is motivated by a lack of trust between private non-financial agents which prevents them from extending trade credit among themselves (i.e. issuing their own IOUs to purchase goods/services on credit). Banks are assumed to have a "multilateral commitment" technology and to exchange their own "multilateral enforceable" IOUs ("bank deposits") for the "unenforceable" IOUs of a nonfinancial agent ("bank loan") who needs to buy on credit (figure 1).

Since banks are trust-worthy in the sense that their liabilities are perfectly enforceable, the seller of the "credit good" accepts them whereas it would not accept the IOUs of its direct (nonbank) trade partner. Operational costs of banks are normalised to zero. Furthermore, since banks are trusted by all agents in the economy, their liabilities are multilaterally enforceable (not only bilaterally), namely they are perfectly liquid, and thus they can circulate as "money". Without loss of generality, all IOUs issued in the economy are one-period IOUs.

The banking sector is composed by a large number of identical banks interacting on three

[8]Note that, if all agents perfectly trusted each other, another option would be for consumers to issue units of credit (IOUs) to purchase goods from each firm, and at the end of each period, these units of credit to be settled among firms and workers (given the return on labor to which they are entitled). In this case an additional perfectly liquid asset such as "money" would play no special role in the economy. This is the very meaning of the cashless limit. As agents do not trust each other however this option is not implementable anymore in the current context.

different perfectly competitive markets: loan, deposit and interbank markets. Equilibrium in the loan market determines the one-period (loan) interest rate that non-financial agents need to pay to banks, whereas equilibrium in the deposit market determines the one period (deposit) interest rate that banks need to pay to (non-financial) bearers of their liabilities issued as a counterpart of loans[9]. Deposits issued by all banks are identical and thus they can be exchanged at par value. Whenever this happens the initial bank issuer of the IOU enters a credit relation with the new bank (figure 2). The interest rate on such "interbank credit" is decided on the interbank market. It is assumed without loss of generality that private credit can only be intermediated via the private banking sector (namely, households and firms cannot exchange their IOUs, or banks' IOUs directly with the ones of the central bank).

3.2 Monetary policy implementation with inside-money

In the cashless basic NK model (Galí (2015), Woodford (2003)), monetary policy controls nominal (and real) interest rates in the economy by setting the price of a one-period government bond in zero net supply. In the "inside-money" economy, the central bank controls the one-period nominal interest rate in the economy by controlling the interbank market rate. It does so by committing (i) to exchange its own IOUs with the IOUs issued by any bank in the economy, and to pay a chosen interest rate ("policy rate") on its IOUs ("deposit facility"), and (ii) to require the same interest rate from the original bank issuer of the IOUs ("borrowing facility"). At equilibrium, no bank has an incentive to require for an interbank market loan an interest rate higher than the policy rate. This can be proved by contradiction. Assume the creditor bank (Bank B in figure 2) asked for an interbank rate higher than the policy rate[10]. Then the debtor bank (Bank A in figure 2) could always exchange the IOU of Bank B for central bank's IOUs and pay instead the (lower) policy rate. In this case Bank B (the creditor bank) would necessarily receive the (lower) policy rate from the central bank[11]. Note that banks in this setup are indifferent between extending credit directly among themselves at the policy rate or making use of the deposit and borrowing facilities of the central bank.

Furthermore, no bank has an incentive to pay an interest on deposits different than the policy rate. If a bank paid a lower deposit rate, then nonbank agents would exchange them for the IOUs (deposits) of another bank. In this case, the initial bank would have to (eventually) pay to the new bank the interbank market rate which is equal to the policy rate. Thus, at equilibrium, all banks necessarily set the deposit rate equal to the policy rate.

[9] Note that the bearers of banks' IOUs may change within the period as deposits are used in transactions. For instance, when households take a bank loan to finance consumption, the initial bearer of the paper is a household, then the IOU is transferred to a firm and then it returns to a household once wages are paid at the end of the period.

[10] Note that the interbank market rate cannot be lower than the policy rate. If this were true, the creditor bank would not lend funds directly to the debtor bank and would prefer to exchange instead the IOU of the debtor bank with the one of the central bank to gain a higher interest rate (the policy rate).

[11] Note how the implementation is in line with the actual functioning of the banking sector: with cash holdings set to zero, if certain banks choose to use the "deposit facility" of the central bank instead of lending their overnight deposits surplus to other banks on the interbank market, other banks in the system will necessarily have to refinance themselves at the central bank via the "borrowing facility".

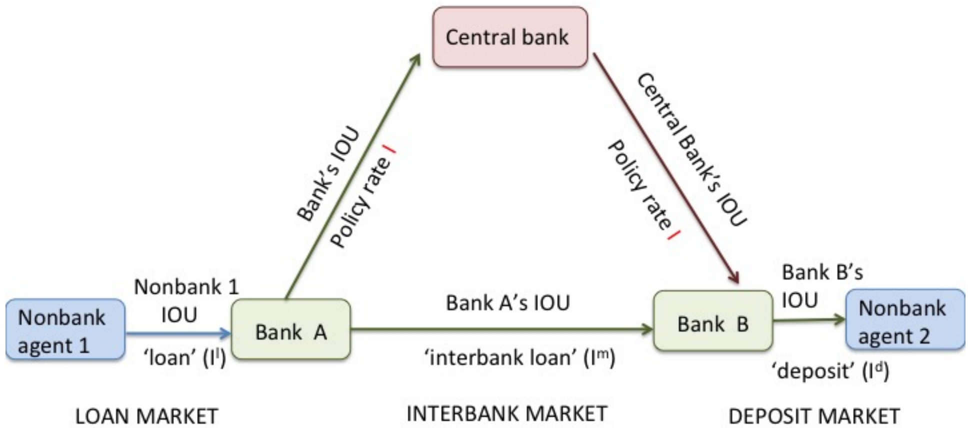

Figure 2: Monetary policy implementation *via* the interbank market

And finally, the loan interest rate equals also at equilibrium the policy rate. If a bank required a loan interest rate higher than the deposit rate (which equals the policy rate), then demand for its services would be zero because there would always exist other banks which can propose one period loans at the (strictly lower) policy rate. Thus, at equilibrium, loan, deposit and interbank markets all clear at a nominal interest rate i_t equal to the policy rate chosen by the central bank.

3.3 A cashless economy *versus* a monetary economy with inside-money

I now tackle the first concern over the lack of monetary fundamentals of cashless NK models by establishing an isomorphism with their monetary versions with liquidity-in-advance constraints and inside-money (bank deposits). In the version presented in this section, households receive their wage at the end of each period, but need to consume at the beginning of the period (households face "cash-in-advance" constraints). In the second version, included in the Appendix on page 18, firms need to finance the wage bill at the beginning of the period before receiving proceeds on sales ("working-capital-in-advance" constraint). Importantly, relatively to the cashless basic NK model, in the "inside-money" versions there is no government bond in zero-net-supply. There is no need for this asset since monetary policy is implemented via the interbank market. Private agents can save instead by investing in one-period bank deposits.

In the first version where the representative household receives the wage at the end of each period, its behaviour is described following the cash-in-advance literature by considering separately a liquidity-in-advance constraint (1), an equation showing its outstanding wealth at the end of the period (2), and a solvency condition (3). The liquidity-in-advance constraint (1) states that *at the beginning of the period* households can pay for consumption goods C_t at price P_t in two ways. One is by using maturing bank deposits $(1 + i_{t-1})D_{t-1}$. The other is by issuing debt to firms via the banking sector, namely by exchanging their own IOUs (which are not enforceable, and hence cannot be used to buy goods on credit) with the ones of banks (which are multilaterally enforceable), and by transferring the latter to firms (as in figure 1).

The second option results in households having a liability towards banks (“households taking a bank loan”), and, banks, in turn, having a liability towards firms (“firms receiving banks’ deposits”). Following convention in the literature, the liabilities of banks towards non-bank agents (“bank deposits”) are denoted by $D_t \geq 0$. Thus, $D_t < 0$ stands for the liabilities of a non-bank agent towards banks (“bank loan”). I distinguish between *new* liabilities of banks towards non-bank agents *created* at the beginning of the period which I denote by D_t^*, and *outstanding* liabilities of banks towards non-bank agents at the beginning of the period which I denote by D_t. For the household, $-D_t^*$ denotes IOUs issued by households to banks at the beginning of period t (“bank loans” contracted at the beginning of the period), whereas D_t denotes outstanding IOUs of banks towards households at the end of period t (outstanding “bank deposits” at the end of t).

The second constraint (2) states that the value of nominal wealth held as bank deposits at the end of the period equals the wage income $W_t L_t$ (transferred in the form of deposits at the end of period, with W_t the nominal wage rate and L_t the number of labor units), firms’ dividends Div_t, the initial amount of deposits $\Big((1+i_{t-1})D_{t-1} + (-D_t^*)\Big)$, net of consumption expenditures $P_t C_t$ and (outstanding) bank credit contracted at the beginning of the period $(-D_t^*)$. Specifically, each household solves

$$\max_{C_t, L_t, D_t, D_t^*} E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \qquad \text{subject to:}$$

$$\mu_t : P_t C_t \leq (1+i_{t-1})D_{t-1} + (-D_t^*) \tag{1}$$

$$\lambda_t : D_t = W_t L_t + Div_t + \Big((1+i_{t-1})D_{t-1} + (-D_t^*)\Big) - P_t C_t - (-D_t^*) \tag{2}$$

$$\lim_{T\to\infty} E_t \Big\{ \beta^{T-t} \frac{U_{c,T}}{U_{c,t}} \frac{D_T}{P_T} \Big\} \geq 0 \tag{3}$$

where C_t is a standard Dixit-Stiglitz consumption index of a continuum of varieties i indexed on the unit interval $C_t \equiv \Big(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \Big)^{\frac{\epsilon}{\epsilon-1}}$ with the associated price $P_t \equiv \Big(\int_0^1 P_t(i)^{1-\epsilon} di \Big)^{\frac{1}{1-\epsilon}}$. The first-order Kuhn-Tucker conditions for a maximum are:

$$\begin{aligned}
C_t : U_{c,t} + \mu_t P_t + \lambda_t P_t &= 0, \\
L_t : U_{l,t} - \lambda_t W_t &= 0, \\
D_t : -E_t\{\beta \mu_{t+1}\}(1+i_t) + \lambda_t - E_t\{\beta \lambda_{t+1}\}(1+i_t) &= 0, \\
D_t^* : \mu_t &= 0, \\
\mu_t : \Big((1+i_{t-1})D_{t-1} - D_t^* - P_t C_t\Big)\mu_t &= 0, \\
\mu_t \leq 0, \ \ (1+i_{t-1})D_{t-1} - D_t^* - P_t C_t &\geq 0, \\
\lim_{T\to\infty} E_t \Big\{ \beta^{T-t} \frac{U_{c,T}}{U_{c,t}} \frac{D_T}{P_T} \Big\} &= 0,
\end{aligned}$$

and the equation showing its outstanding wealth at the end of the period (2).

Note that in this setup the liquidity-in-advance constraint is always slack ($\mu_t = 0$) i.e. it does not constrain household's choice. The reason is that in the 'inside-money' economy households can costlessly spend in advance their period wage income by means of bank credit. Thus, after combining previous equations, we can describe the behaviour of the representative household using the same equations as in the cashless basic NK model (i.e. standard Euler, labor supply and household budget constraint equations):

$$\beta(1+i_t)E_t\Big\{\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\Big\} = 1,$$
$$-\frac{U_{l,t}}{U_{c,t}} = \frac{W_t}{P_t},$$
$$W_tL_t + Div_t + (1+i_{t-1})D_{t-1} = P_tC_t + D_t.$$

The supply-side of the economy is also identical to the one in the flexible price version of the basic cashless NK model (e.g. Galí (2015), Chapter 3). Specifically, it is composed by a continuum of diversified firms in monopolistic competition with a Cobb-Douglas production technology. The input market (here, labor market) is competitive and firms act as price-takers. Firms need to pay inputs (here, wages) by the means of a perfectly liquid financial asset. They cannot issue such an asset, but they can use banks IOUs (banks' deposits) for this purpose (either by exchanging its own IOUs with banks' IOUs, or by using outstanding holdings of such assets). Since in this version firms pay wages at the end of the period, and sell their products within the period, they face no binding liquidity- constraints. Specifically, the IOUs received from selling goods are used to pay workers and shareholders. Thus, the problem of firm i writes

$$\max_{P_t(i),Y_t(i),L_t(i)} P_t(i)Y_t(i) - W_tL_t(i) \qquad \text{subject to:}$$
$$Y_t(i) = A_tL_t(i)^{1-\alpha}$$
$$Y_t(i) = \Big(\frac{P_t(i)}{P_t}\Big)^{-\epsilon}Y_t$$

where $Y_t(i)$ stands for output of firm i and A_t is an exogenous productivity process. The behaviour of firm i is thus described by

$$P_t(i) = \mathscr{M}\frac{W_t}{(1-\alpha)A_tL_t^{-\alpha}(i)}$$
$$Y_t(i) = A_tL_t(i)^{1-\alpha}$$
$$Y_t(i) = \Big(\frac{P_t(i)}{P_t}\Big)^{-\epsilon}Y_t$$

Standard market clearing conditions apply on goods and labor markets. At equilibrium, from representative household's budget constraints (1) and (2), we get $D_t = (1+i_{t-1})D_{t-1}$. For $D_{-1} = 0$, this implies $D_t = 0, \ \forall t$. Finally, nominal determinacy is achieved, for instance, by having the central bank follow a policy rule which ensures equilibrium uniqueness.

We can thus conclude that the equations describing the dynamics of (real or nominal) variables in the model are identical to the ones in the cashless basic NK model (here, its flexible price version) despite the liquidity-in-advance constraints of private non-financial agents motivated by the decentralization of trade, and the lack of trust between these agents (i.e. lack of reenforceability of trade credit among them). Otherwise stated, the monetary version with inside-money issued within a perfectly competitive banking sector is isomorphic to the cashless one. Furthermore, I show in the Appendix 7.1 on page 18 that if firms need to pay instead wages at the beginning of the period in advance of sales (i.e. they have a"working capital in advance constraint"), the same isomorphism between the version with inside-money and the cashless version emerges.

3.4 Inside-liquidity banking theory and NK models

In this section I tackle the second concern regarding the way banks are modeled when explicitly included in NK models, namely the lack of reference to their role in money creation. I take as a reference the model in Bernanke, Gertler and Gilchrist (1999) (hereafter, BGG (1999)), and I compare the equilibrium dynamics of the original model where financial intermediaries take deposits from households and lend them to entrepreneurs for investment purposes, with a version where they behave in line with the inside-liquidity banking theory. The choice of this model is without loss of generality. Even when financial intermediaries are explicitly identified with "banks" (e.g. Gertler and Kiyotaki (2011), Gertler and Karadi (2011)), their behaviour is described in the same manner as in BGG (1999). To focus strictly on the impact of the two different approaches to modeling banks' behaviour on equilibrium dynamics, I abstract from financial frictions and nominal price rigidities.

The setup is an extension of the one described in the previous section with production run by perfectly competitive risk-neutral entrepreneurs who use both labor and physical capital as inputs, and finance physical capital both with their own funds (retained earnings) and loans from a financial intermediary.

Non-financial agents Each period, entrepreneurs make two types of choices: a production decision given their outstanding physical capital, and a capital investment decision for production in the following period. As in BGG (1999), firms resell and rebuy each period their entire capital stock on the market. Namely, given capital level K_t chosen in the previous period, each entrepreneur i solves

$$\max_{Y_t(i),L_t(i)} P_t(i)Y_t(i) - W_t L_t(i) \qquad \text{subject to:}$$
$$Y_t(i) = A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha}$$

The production technology is assumed to exhibit constant returns to scale. Production choices by each entrepreneur i given its capital stock are described by

$$P_t(i) = \frac{W_t}{(1-\alpha)\frac{Y_t(i)}{L_t(i)}},$$
$$Y_t(i) = A_t K_t(i)^{\alpha} L_t(i)^{1-\alpha}.$$

Importantly, note that in contrast to the model in the previous section, sales income now comes from *both* households (consumption) and firms (investment).

Following BGG (1999), capital investment decisions are taken at the end of each period (given expected return), and risk neutral entrepreneurs are willing to absorb the associated macroeconomic risk. Capital is homogenous, namely newly produced capital units within the period have the same value as older vintages and thus sell at the same price Q_t.

The whole stock of capital is financed each period by bank credit repaid with interest in the following period (short term debt) and entrepreneurial net worth (to be defined latter). Entrepreneurs invest in capital goods until the expected nominal return $E_t\{R^k_{t+1}\}$ equals the gross nominal loan interest rate. The latter equals the one-period gross nominal interest rate $1 + i_t$ since aggregate risk is borne by firms and thus loans are risk-free for banks[12]:

$$E_t\{R^k_{t+1}\} \equiv E_t\left\{\frac{\frac{\alpha P_{t+1}(i)Y_{t+1}(i)}{\mathscr{M}K_{t+1}(i)} + Q_{t+1}(1-\delta)}{Q_t}\right\} = 1 + i_t$$

where δ denotes the depreciation rate and $\frac{\alpha P_{t+1}(i)Y_{t+1}(i)}{\mathscr{M}K_{t+1}(i)}$ is the expected marginal (nominal) return of capital.

As in BGG (1999), entrepreneurs have finite lives and a constant survival probability γ to the next period. The birth rate of entrepreneurs is such that the fraction of agents who are entrepreneurs is constant. This assumption avoids the case where the entrepreneurial sector ultimately accumulates enough wealth to be fully self-financing. Entrepreneurs dying in period t are not allowed to invest in capital, but instead simply consume their retained earnings. Furthermore, total labor input L_t is a composite index of household labor L^h_t, and "entrepreneurial labor", L^e_t, namely $L_t = (L^h_t)^{\Omega}(L^e_t)^{1-\Omega}$, with entrepreneurial labor supplied inelastically and total entrepreneurial labor normalized to unity. Entrepreneurial labor is used to make new firms start with some initial net worth. End-of-period net worth of all entrepreneurs surviving to the next period equals

$$N_{t+1} = \gamma\Big(Q_t K_t - (1+i_{t-1})(Q_{t-1}K_t - N_t) - \delta K_t Q_t\Big) + W_t + (P_t Y_t - W_t L_t) \quad (4)$$

where $Q_t K_t$ is the market value of outstanding capital holdings, $(1 + i_{t-1})(Q_{t-1}K_t - N_t)$ is the gross nominal interest rate paid on the loan taken in the previous period to finance capital, $\delta K_t Q_t$ is

[12] I assume that the return to capital is sensitive only to aggregate risk since idiosyncratic risk does not play any particular role in the absence of financial intermediation frictions.

the market value of depreciated capital goods, W_t is the entrepreneurial wage. Since firms are now owned by entrepreneurs, households receive no dividends (i.e. $Div_t = 0$). The aggregate physical capital dynamics are described by[13]:

$$K_{t+1} = (1-\delta)K_t + \mathscr{I}_t \tag{5}$$

where $\mathscr{I}_t$ is net investment in period t, and δ is the capital depreciation rate. Households behave exactly as in the model used in the previous section.

"Banks" – mainstream versus inside-liquidity theory In mainstream macroeconomic models such as BGG (1999) savings of households are channelled through financial intermediaries to fund the acquisition of physical capital goods by firms[14]. Even when financial intermediaries are explicitly identified with banks, their behaviour is described in the same manner (e.g. Gertler and Kiyotaki (2011), Gertler and Karadi (2011))[15]. As shown next however, this is without loss of generality since when the (more realistic) inside-liquidity theory of banking is applied instead, despite its distinct narrative, equilibrium dynamics of resulting models are identical.

Specifically, consider the case of firms willing to purchase capital goods in the inside-liquidity banking theory case. The potential sellers of such goods are other firms in the economy, namely the producers of new capital goods and owners of older capital vintages willing to liquidate them. If firms trusted each other, they would extend trade credit directly among themselves i.e. they would issue one period IOUs to the seller of capital goods equal to the value of capital goods augmented interest (equal in equilibrium to the expected capital return)[16].

However, since this is not the case, in each such trade credit relation a bank acts as an intermediary by exchanging its own one period IOUs ('bank deposits') for the one period IOUs of the 'buyer firm' ('loan'), banks' IOUs being the ones given to the 'seller firm' in exchange for capital goods (figure 1). As a result entrepreneurs will issue (among themselves) an aggregate amount of bank deposits equal to the market value of the fraction of capital goods externally financed, namely $\left(Q_t K_{t+1} - N_{t+1}\right)$. In the aggregate, these deposits are received as (current) sale revenue and are used both to pay (current) wages and, as part of internal funds, to buy investment goods.

Combing the representative household budget constraint (2) with the goods market clearing condition $Y_t = C_t + C_t^e + \mathscr{I}_t$ and setting $Div_t = 0$, we obtain that the following relation is satisfied

[13] BGG (1999) additionally included increasing marginal adjustment costs in the production of capital to allow a variable price of capital. To ease exposition, I abstract from such costs since they are not relevant for the current argument.

[14] BGG (1999) p.1349: 'The entrepreneur borrows from a financial intermediary that obtains its funds from households'.

[15] Sometimes banks' are modelled as facing themselves frictions in financing their loan portfolio and thus their own net worth may impact real economic dynamics (e.g. Gertler and Kiyotaki (2011), Gertler and Karadi (2011)).

[16] In BGG (1999), entrepreneurs are risk neutral, and hence willing to absorb the macroeconomic risk from capital investment.

in equilibrium:

$$W_tL_t^h + (1+i_{t-1})D_{t-1} = P_tY_t - P_tC_t^e - P_t\mathscr{I}_t + D_t. \tag{6}$$

By further replacing the expression of $(P_tY_t - W_tL_t^h)$ from the equation above in the expression of aggregate net worth of entrepreneurs (4), and using $W_tL_t = W_tL_t^h + W_t$, it yields:

$$\begin{aligned}N_{t+1} &= \gamma\Big(Q_tK_t - (1+i_{t-1})(Q_{t-1}K_t - N_t) - \delta K_tQ_t\Big) + W_t + (1+i_{t-1})D_{t-1}\\ &+ P_tC_t^e + P_t\mathscr{I}_t - D_t - W_t\\ &= \gamma\Big(Q_tK_t - (1+i_{t-1})(Q_{t-1}K_t - N_t) - \delta K_tQ_t\Big) + (1+i_{t-1})D_{t-1}+\\ &+ P_tC_t^e + P_t\mathscr{I}_t - D_t\end{aligned}$$

After replacing the expression of aggregate entrepreneurial consumption $P_tC_t^e = (1-\gamma)\Big(Q_tK_t - (1+i_{t-1})(Q_{t-1}K_t - N_t) - \delta K_tQ_t\Big)$, the relation above becomes:

$$N_{t+1} = \Big(Q_tK_t - (1+i_{t-1})(Q_{t-1}K_t - N_t) - \delta K_tQ_t\Big) + (1+i_{t-1})D_{t-1}+$$
$$+P_t\mathscr{I}_t - D_t$$

Further using the expression of net capital investment $\mathscr{I}_t$ from the law of motion of capital (5) and that $Q_t = P_t$ yields:

$$\begin{aligned}(Q_tK_{t+1} - N_{t+1}) &= (1+i_{t-1})(Q_{t-1}K_t - N_t) - (1+i_{t-1})D_{t-1} + D_t\\ (Q_tK_{t+1} - N_{t+1} - D_t) &= (1+i_{t-1})(Q_{t-1}K_t - N_t - D_{t-1})\end{aligned}$$

For $Q_{t-1}K_t - N_t - D_{t-1} = 0$, this implies

$$Q_tK_{t+1} - N_{t+1} = D_t, \ \ \forall t$$

or, the value of capital financed by entrepreneurs via the banking sector, equals in equilibrium the value of one period deposits held by households.

Thus, when applying inside-liquidity banking theory, end-of-period savings of households D_t (in terms of bank deposits) also equate in equilibrium the market value of investment externally financed by entrepreneurs via the banking sector $Q_tK_{t+1} - N_{t+1}$. So, even though the two approaches to the role of banks in the economy have distinct narratives, these narratives imply isomorphic equilibrium dynamics[17].

We may thereby conclude that *under the assumptions of mainstream macroeconomic models*,

[17] In particular, note that following the inside-liquidity narrative, it is the gross investment level (i.e. the value of capital in the economy) which determines the end of period level of outstanding savings, whereas it is the other way round in the alternative (conventional) case.

where the liquid nature of banks' liabilities does not play any specific role, the explicit monetary role of banks is irrelevant for equilibrium dynamics. Thus, the abstraction made is without loss of generality. For completeness, the equations describing the equilibrium dynamics of the model specification with capital investment encompassing the narratives of both banking theories are summarized in the Appendix 7.2 on page 19.

4 On the relevance of banks' monetary role

These equivalence results do not imply however that the monetary role of banks should generally be thought as irrelevant for monetary policy within the NK paradigm. To make this point, I discuss in this section some cases where modeling it explicitly, as we do in our current analysis, is consequential.

To begin with, the inside-money version of the basic NK model presented in section 3.3 could serve as an analytical exposition for the cash abolishment proposal made by Rogoff (2017) to eliminate the zero lower bound constraint (and associated inefficiencies)[18]. Specifically, it allows one to get an intuition on how monetary policy could be implemented if the central bank stopped issuing banknotes and coins and why the zero lower bound would become irrelevant in such a world[19]. Furthermore, it helps highlighting how negative nominal interest rates are only a convention strictly related to the unit of account and there is no conceptual difference between positive and negative nominal interest rates. In the words of Rogoff (2017), negative rate policy would just be "central banking business as usual, namely cutting interest rates in negative territory would work the same way as interest rate cuts in positive territory".

Importantly, the inside-money versions described in this paper provide a first explicit modeling within the NK framework of how a central bank could implement its policy if it stopped supplying *non-interest liabilities*. Standard cashless setups feature a zero-lower bound (ZLB) constraint because, as long as policy rates are strictly positive, such liabilities ("cash") are supplied by the central bank even though they are not held in equilibrium (e.g. Woodford (1999), Woodford (2003), Chapter 1)[20]. Otherwise stated, the term "cashless" strictly refers to the equilibrium outcome when

[18] I thank my PhD co-advisor Jordi Galr bringing to my attention the existence of this book and of a potential link to the analysis in my paper.

[19] Note that in the current setup with inside-liquidity created within the banking sector monetary policy faces no constraints in setting the policy rate to negative values.

[20] (i) Woodford 1999 page 34: "note that the equations [of the cashless model] are not simply the cashless limit of the equilibrium conditions of a monetary economy; they are also the equilibrium conditions that must be satisfied by the real interest rate and real financial wealth in a completely non-monetary economy; thus they could easily be derived by abstractly entirely from the use of money in transactions. *The only reason that I have described the system consisting of these equations together with the policy rules as determining the price level in the cashless limit of a monetary economy- rather than simply saying that they describe price level determination in an economy where cash is not needed for transactions is that it is not clear that a central bank should have any way of implementing the policy rule when money is not used at all, even though it can implement such a rule in a monetary economy no matter how close to zero alfa may be.*" (ii) Woodford 1999 page 26: "the central bank controls the rate on the market for short term nominal debt by staying ready to exchange public debt for money in arbitrary quantities at the price that it has decided upon. It is not possible for the central bank to bring about an interest rate $R < 1$ since this would be

nominal interest rates are strictly positive. Once the policy rate reaches the ZLB however, the central bank loses in these standard setups its power to influence nominal (and real) interest rates in the economy i.e. the model economy enters a "liquidity trap".

Second, the explicit incorporation of "inside-money" in the NK framework makes clear that interpreting "money" in extensions with "money-in-the-utility" (MIU) differently than "monetary base (M0)", namely M1, M2 or M3 (e.g. M1 in Cooley and Hansen (1989, 1991), M2 in Ireland (2004)), is problematic. Note that in the basic NK model with inside-money (both its cash-in-advance and working capital-in-advance versions) agents use in transactions exclusively monetary aggregates whose real counterpart is (the privately issued fraction of) M1. Nevertheless, the setup is isomorphic to the "cashless" version of the model (which is not the case for MIU specifications). Same logic would apply for M2 and M3. Thus, as pointed out by Woodford (2003) pp. 117, "money" should be strictly interpreted in these frameworks as central bank liabilities in positive net supply (i.e. the monetary base M0, "outside-money" as opposed to "inside-money").

Third, and most importantly, one could imagine NK setups populated by several types of financial intermediaries, where the high degree of multilateral enforceability of banks' liabilities is relevant for the transmission of monetary policy and for macroeconomic dynamics and allocation. Such analyses may uncover insights for instance regarding differences in monetary policy transmission and in the responses of macroeconomic variables to shocks at business cycle frequencies for economies with different financial structures such as the bank-based case in the Euro Area and the market-based one in the United States[21]. One promising starting point to build such an extension could be the model in Kiyotaki and Moore (2018) which explicitly takes into account that "fiat money" issued by the central bank is more liquid than equity. Specifically, one could replace "fiat outside money" with "bank inside-money", and add in the analysis frictions specific to financial intermediation via the banking sector.

5 Conclusions

I addressed in this paper two related concerns expressed by economists working at monetary policy institutions regarding the lack of explicit account of banks' monetary role in NK models, namely (i) the lack of reference to bank deposits in cashless versions of these models, and (ii) the lack of explicit account of banks' monetary role once included in this framework. I tackled the first concern by showing that the cashless specification is isomorphic to monetary versions with bank deposits

inconsistent with equilibrium owing to the arbitrage opportunity that it would create". (iii) Woodford 2003 Chapter 1.2. page 75: "The function [of the interest rate target rule] is assumed to be nonnegative on the grounds that it is not possible for the central bank to drive nominal interest rates to negative levels. I assume that, as under typical current arrangements, the holders of central bank balances have the right to ask for currency in exchange for such balances at any time and that it is infeasible to pay negative interest on currency. Hence an attempt to pay negative interest on central bank balances would lead to zero demand for such balances and a market overnight interest rate of zero rather than a negative overnight interest rate".

[21] This research idea was first brought to my attention long before writing this paper during the time I was working as a research analyst in the Monetary Policy Strategy division of the European Central Bank by Jens Eisenschmidt, Principle Economist in the division.

(inside-money) used in transactions, and the second one, by showing that banks' monetary role is irrelevant for equilibrium dynamics under the assumptions of standard models. In the end, with the help of some examples, I pointed out however that these results do not imply that banks' monetary role should generally be thought as irrelevant for monetary policy analyses within this paradigm. To my knowledge, there are no extensions of the NK framework where banks behaviour is modeled in line with inside-liquidity theory and banks' role in the supply of liquidity is relevant for the equilibrium allocation. Exploring this line of research may however uncover new interesting findings regarding the transmission and optimal design of monetary policy.

6 References

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7 Appendix

7.1 Specification with inputs paid in advance of sales

This section discusses the alternative specification where firms need to pay working capital (here, wages) in advance of sales as in Christiano and Eichenbaum (1992). In this setup the constraints faced by the household slightly change since labor income is received at the beginning of the period: the liquidity-in-advance constraint (1) and the equation showing its outstanding wealth at the end of the period (2) are now

$$\mu_t : P_t C_t \leq (1 + i_{t-1}) D_{t-1} + W_t L_t - D_t^* \tag{7}$$

$$\lambda_t : D_t = Div_t + \left((1 + i_{t-1}) D_{t-1} + W_t L_t - D_t^* - P_t C_t\right) + D_t^* \tag{8}$$

Note that only the expression of the optimality condition with respect to labor changes (i.e. $U_{l,t} - W_t(\mu_t + \lambda_t) = 0$). However, since $\mu_t = 0$, households' behaviour is eventually described by the same equations as in the case presented in section 3.3. Households do face, however, liquidity-in-advance constraints (even though slack), hence they purchase goods in exchange of bank deposits.

Firms use bank loans to finance production in advance of sales. Specifically, they exchange their IOUs ("loans") with banks' IOUs ("deposits") which they use to pay workers. Until the end of the period they sell all goods in exchange of bank deposits. Thus, their decision problem *in terms of financial wealth at the beginning of next period* writes[22]:

$$\begin{aligned} \max_{P_t(i), Y_t(i), L_t(i)} (1 + i_t) P_t Y_t - (1 + i_t) W_t L_t \qquad & \text{subject to:} \\ Y_t(i) &= A_t L_t(i)^{1-\alpha} \\ Y_t(i) &= \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t \end{aligned} \tag{9}$$

So, again, their behaviour is described by the same equations as in the cashless version despite the liquidity-in-advance constraints they face.

[22]We follow the approach in the "cost-channel literature" and we assume that firms take one period loans. Note that if we assumed instead that they could repay their loan at the end of the period after receiving proceeds on sales (namely, the case of an intra-temporal loan), the same isomorphy would emerge in the case with inside-money.

Finally, it is important to recall that in contrast to the current inside-money setup, in models with outside-money, the "working-capital in advance" constraint generates an additional "cost channel" of monetary policy transmission (e.g. Ravenna and Walsh (2006), De Fiore and Tristani (2013)) with respect to the basic (credit-frictionless) New-Keynesian model. As a result, the version with "working-capital in advance" is not isomorphic anymore with the latter. This is because firms receive the proceeds on sales in terms of *non-remunerated central bank liabilities*, namely they do not receive any intertemporal ("overnight") interest on P_tY_t (as it is the case in the setup with 'inside-money', equation (9)).

7.2 Equations of the model with capital investment

$$\beta(1+i_t)E_t\Big\{\frac{U_{c,t+1}}{U_{c,t}}\frac{P_t}{P_{t+1}}\Big\} = 1$$

$$-\frac{U_{l,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_tK_{t+1} - N_{t+1} = D_t$$

$$N_{t+1} = \gamma\Big(Q_tK_t - i_{t-1}(Q_{t-1}K_t - N_t) - \delta K_tQ_t\Big) + W^e_t + (P_tY_t - W_tL_t)$$

$$W^e_t = W_t$$

$$C^e_t = (1-\gamma)\Big(Q_tK_t - i_{t-1}(Q_{t-1}K_t - N_t) - \delta K_tQ_t\Big)$$

$$Y_t = C_t + C^e_t + \mathscr{I}_t$$

$$(1+i_t)Q_t = E_t\Bigg\{\frac{\alpha P_{t+1}Y_{t+1}}{\mathscr{M}K_{t+1}} + Q_{t+1}(1-\delta)\Bigg\}$$

$$K_{t+1} = (1-\delta)K_t + \mathscr{I}_t$$

$$(1-\alpha)\frac{Y_t}{L_t} = \mathscr{M}\frac{W_t}{P_t}$$

$$Y_t = A_tK^{\alpha}_tL^{1-\alpha}_t,\ A_t \text{ exogenous}$$

$$Q_t = P_t$$

$$(1+i_t) = \Big(\frac{P_{t+1}}{P_t}\Big)^{\phi}, \phi > 1$$